

Dovecotes Primary School

Calculation Policy



Our Vision

At Dovecotes Primary School, we believe that children should be introduced to the processes of calculation in ways which: suit their individual learning needs; take into account their prior understanding; and provide a level of challenge to all pupils.

When children leave at the end of Year 6, they will have acquired the skillset, knowledge and understanding to choose the most appropriate method (mental, use of jottings, formal written) to solve a problem. We encourage children to appreciate that mental and written methods complement one another and should not be seen as separate entities.

We use a selection of schemes and resources (White Rose Maths, Abacus, Numicon etc.) and adapt them, where necessary, to meet the needs of our pupils. Subject areas are grouped into blocks and each block begins with a Cold Task.

Cold Tasks assess the pupils against each of the age-related expectations and allow teachers to adjust their planning to meet the needs which are identified as a result of these assessments. The results are also used to plan interventions and ensure progress can be measured on a lesson-by-lesson basis.

The purpose of this document is to ensure a standardised, whole-school approach to teaching and learning. It indicates the addition, subtraction multiplication and division age-related expectations for each year group and models the methods taught at Dovecotes to allow children to achieve them. Methods are organised into 'concrete', 'pictorial' and 'abstract' approaches which can be used to meet individual learning styles. Age-related expectations are highlighted to show whether an expectation is working towards, working at or working beyond the expected standard of a child in that year group.

The policy also demonstrates the mathematical learning journey children will take at Dovecotes and the progression of skills, knowledge and understanding from Year 1 to Year 6.

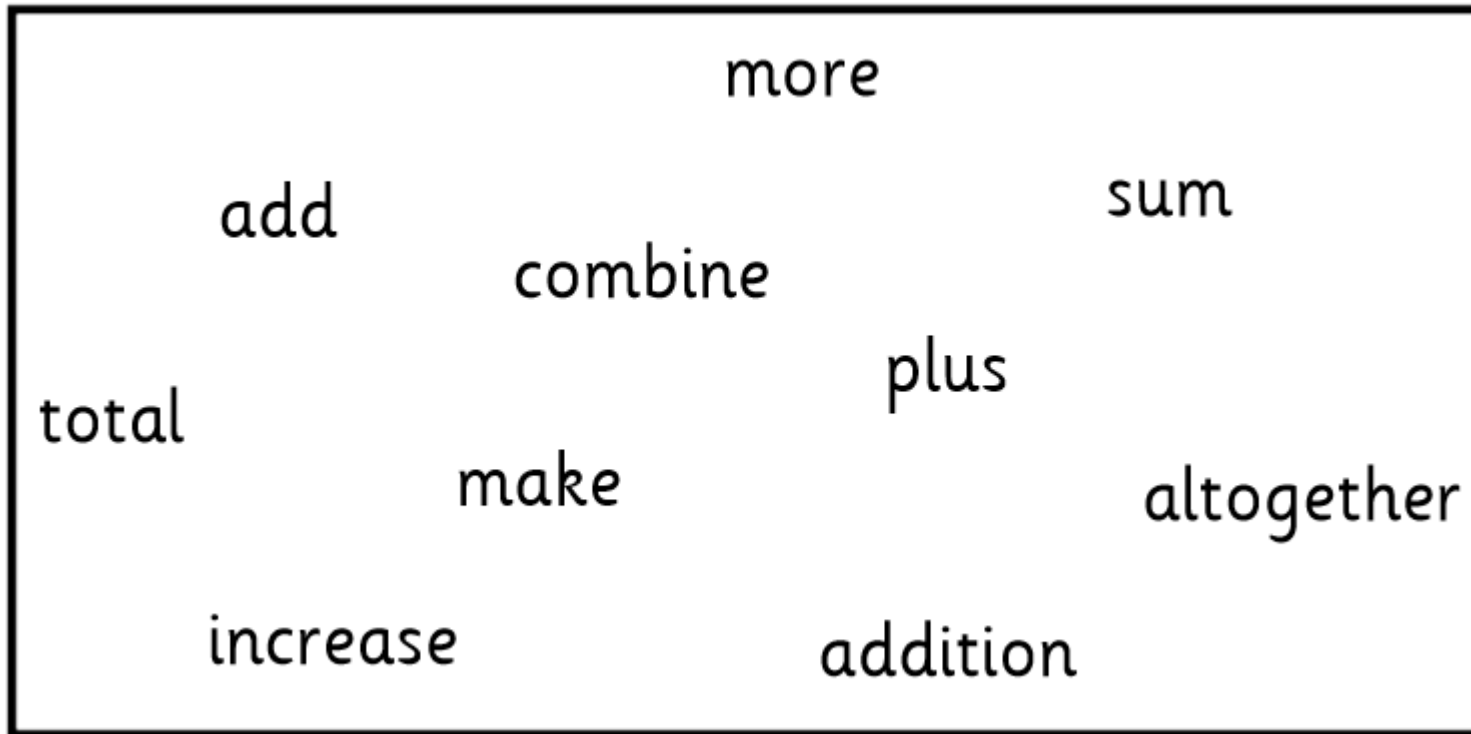
May 2020

Addition



$$\underset{\text{Addend}}{8} + \underset{\text{Addend}}{3} = \underset{\text{Sum or Total}}{11}$$

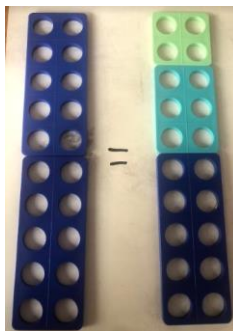
Key Vocabulary



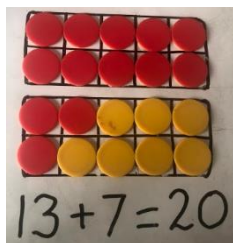
Year 1: Addition

Age-related expectation	Concrete	Pictorial	Abstract
Add 2 single digits up to 20.	Combining two parts using objects to create the whole (sum). Starting at the larger addend and counting on to arrive at the sum. Use Numicon to represent addends and convert to a teen number if necessary. Ten frames with different-coloured counters to show each addend.	Drawing representations of each addend and combining for the sum. Use of a number line. Starting at the greater addend and counting on to arrive at the sum. Using the part-whole model with drawn representations of addends and sum. Drawn representation of a ten frame. Use of squares in maths books encouraged.	Use of a part-whole model, with numerals, to show two addends added to make the sum. Complete the number sentence and then rearrange to show the calculation in different orders.

Answer addition
number bonds to 20.



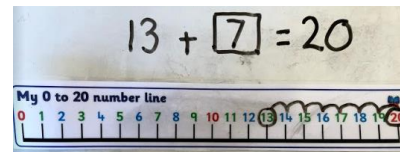
Create 20 using
Numicon shapes.
Show 20 in
different ways (16
+ 4 or 10 + 6 + 4).



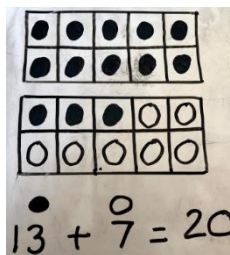
Use pairs of ten
frames to show
different ways of
making 20 with sets
of counters.



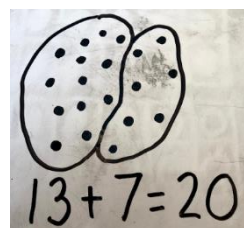
Use differently-coloured cubes to make a "bar" the
same length as 20 cubes. How many of each colour
were used?



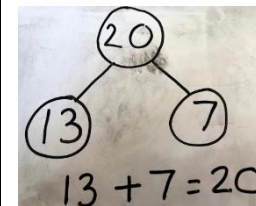
Use of a number line. Count on from any number up
to 20 to identify bonds to 20.



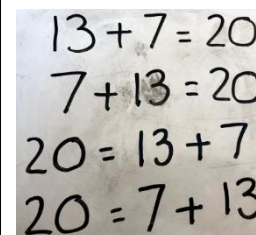
Use pairs of ten frames
to show different ways
of making 20 with
drawn representations.



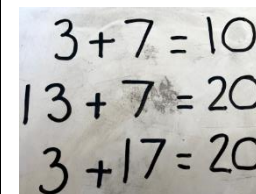
Organise 20
pictorial
representations of
ones into 2 groups
to identify a bond.



Use of a part-whole
model, with
numerals, to show
two addends added
to make 20.

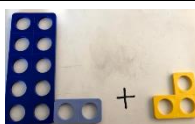


Complete the bond
to 20 and then
rearrange to show
the calculation in
different orders.

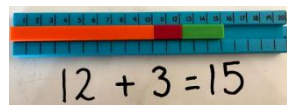


Use of number
bonds to 10 to
make bonds to 20.
Identify adding 10
to either addend.

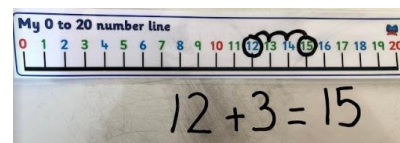
Add a single-digit
number to a two-
digit number up to
20.



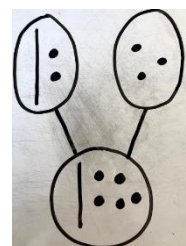
Combining Numicon
shapes to make 2-
digit and 1-digit
numbers and then
combine again to
make the sum.



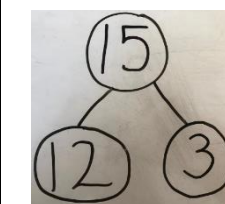
Combine Cuisenaire rods on number tracks to arrive
at the sum.



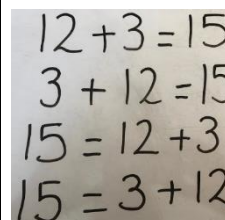
Use of a number line. Starting at the greater addend
and counting on to arrive at the sum.



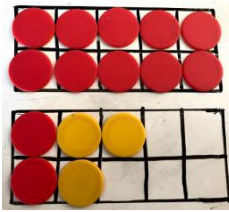
Use a part-whole
model and drawn
representations of
Dienes to combine
two addends to
arrive at the sum.



Part-whole model to
show addends being
combined to make
the sum. Use of
written numerals.



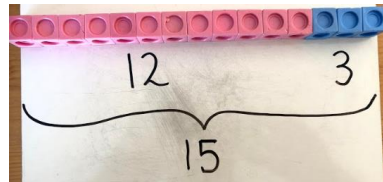
Complete the
number sentence
and then rearrange
to show the
calculation in
different orders.



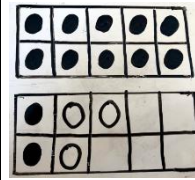
Ten frames with different-coloured counters to show each addend.



Use of bead strings to start at the larger addend and add on to arrive at the sum of both numbers.



Combining physical objects which represent each addend and then counting to find the sum.

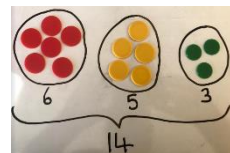
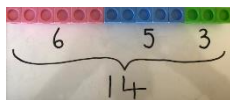


Use pairs of ten frames to add a 1-digit number to a 2-digit number using drawn representations of each addend.

12 add 3 equals 15
3 more than 12 is 15
12 plus 3 equals 15
15 is 12 add 3

Promote the use of appropriate mathematical vocabulary. Provide word mats and allow children to present addition number sentences in verbal and written forms.

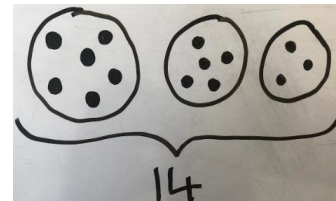
Add 3 single digits up to 20.



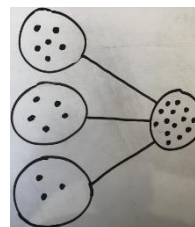
Combine three sets of physical objects (acting as addends) and count to arrive at the sum.



Use of Cuisenaire rods and a labelled number track to combine three addends.



Creating drawn representations of three addends and then counting to find the sum.



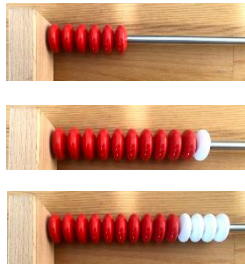
Using the part-whole model with drawn representations of addends and sum.

$6 + 5 = 11$
 $11 + 3 = 14$
so
 $6 + 5 + 3 = 14$

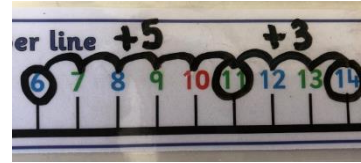
Add each addend in turn to arrive at the sum. Use numerals and mathematical symbols.

$6 + 5 + 3 = 14$
 $6 + 3 + 5 = 14$
 $5 + 6 + 3 = 14$
 $5 + 3 + 6 = 14$
 $3 + 6 + 5 = 14$
 $3 + 5 + 6 = 14$

Rearrange the number sentence in as many ways as possible. Show the order which numbers are added does not change the sum.



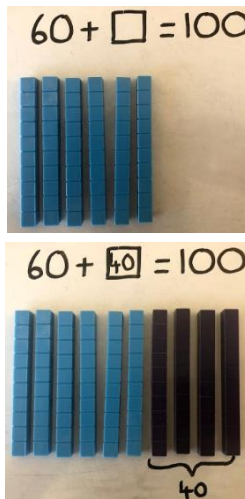
Counting on from the largest number in two steps, one for each further addend.



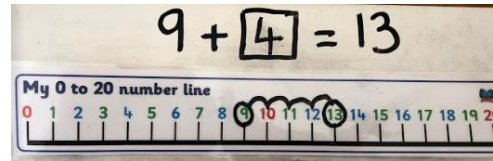
Use of a number line. Start at the largest addend then add the other addends to arrive at the sum.

$$\begin{array}{r} 6 + 5 + 3 \\ \quad \swarrow \searrow \\ \quad 4 \quad 1 \\ 6 + 4 = 10 \\ 1 + 3 = 4 \\ 10 + 4 = \underline{14} \end{array}$$

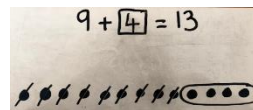
Use knowledge of bonds to 10 to add using mental methods. Written jottings to be encouraged.



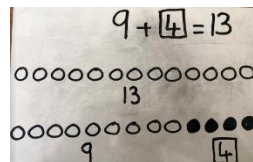
Counting on using apparatus to arrive at the sum.



Use of a number line. Count on from the known addend to the sum. The number of jumps will reveal the missing addend.



Use a drawn representation of the sum and separate it into the known and unknown addend.



Drawn representation of the 'bar' approach to creating equal amounts.

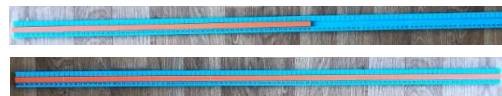
$7 + 5 = 12$

"If I start at 7 and count in ones, I need to count 5 times to get to 12."

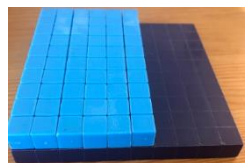
Promote the use of appropriate mathematical vocabulary. Children to verbally explain their workings when finding the missing number in an addition number sentence.

$$\begin{array}{r} 7 + 5 = 12 \\ 12 - 7 = 5 \end{array}$$

Apply knowledge acquired through concrete and pictorial methods to rearrange number sentences (including using the inverse).



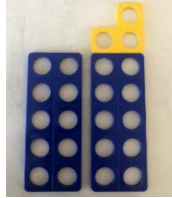
Use of a number track and Cuisenaire rods. Add rods until the sum is reached to identify the missing number.



Where 100 is the sum, use a 100 Dienes and lay Dienes on top to identify the addends.

Solve missing number problems that involve addition up to 100, using apparatus if necessary.

$$16 + \square = 23$$



$$16 + \square = 23$$



$$16 + \boxed{7} = 23$$



Use Numicon to build the sum, place the known addend on top and the space left over will reveal the missing addend.

$$9 + \square = 13$$

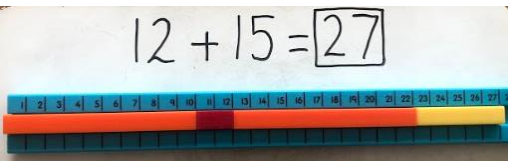
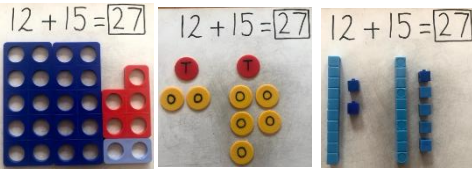
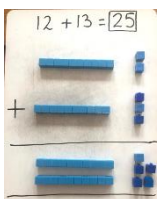
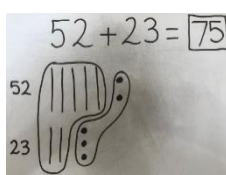
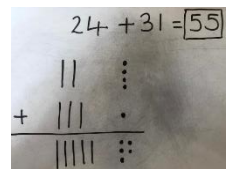
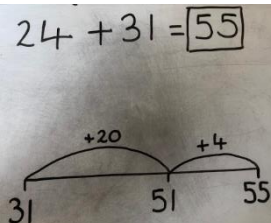
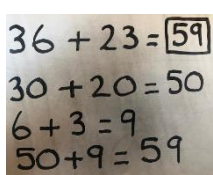
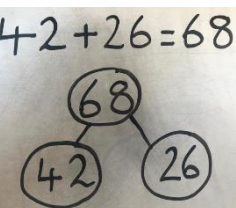
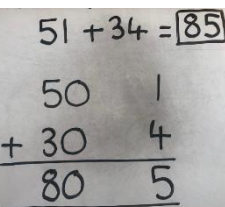
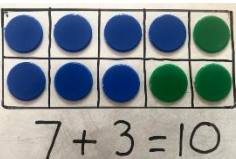
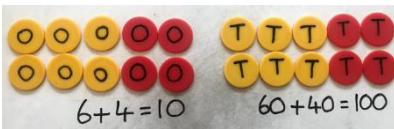
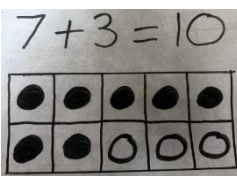
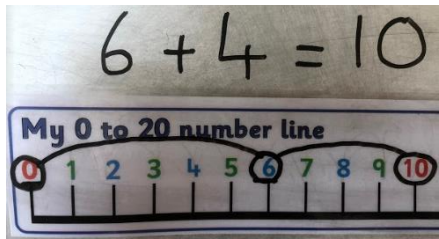
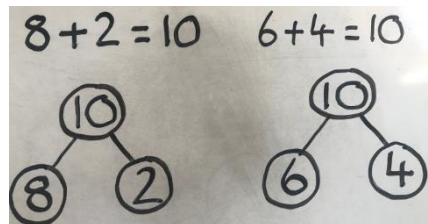
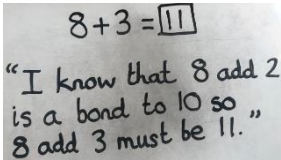


$$9 + \boxed{4} = 13$$

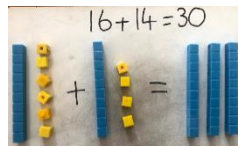


Use cubes to create a 'bar' the length of the sum. Above, build a bar of the know addend and then add cubes so both bars are equal. The cubes added will reveal the missing addend.

Year 2: Addition

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Add 2 two-digit numbers where no regrouping is required (no crossing tens).	  Use a selection of physical manipulatives to combine tens and ones from 2 addends to create the sum.  Organise Dienes in a column format to show tens and ones being added and then recombined.	 Use drawn representations of tens and ones to combine addends to arrive at the sum.  Organise drawn Dienes in a column format to show tens and ones being added and then recombined.  Use a number line to count on from the larger addend to arrive at the sum.	 Partition 2-digit numbers into tens and ones, add and then recombine to find the sum.  Add 2-digit numbers and then present in number sentences and part-whole models.  Partition numbers and add using the expanded column method for addition.
Recall at least four of the six number bonds to 10 and reason about associated facts.	 Use Numicon, overlaying pairs of shapes on top of the 10 shape to reveal bonds to 10.  Ten frames with different-coloured counters to show each addend up to 10.  Use physical representations of ones and tens to model bonds to 10 and associated bonds to 100	 Use a drawn ten frame with different symbols for each addend to show bonds to 10.  Draw pairs of jumps on a number line to 10 to show different number bonds.	 Present number bonds to 10 in number sentences and using part-whole models.  Use appropriate vocabulary to identify bonds to 10 and other known facts using this knowledge.

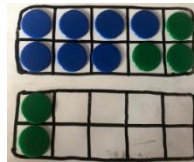
Add 2 two-digit numbers using an efficient strategy, explaining verbally.



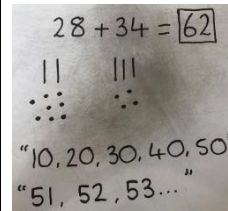
Use physical manipulatives to add pairs of 2-digit numbers. Where the ones add to more than 9, exchange for an additional ten and ones.



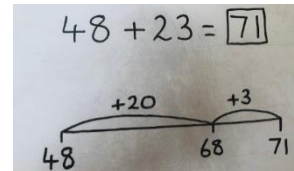
Count on using a bead string, starting at the larger addend.



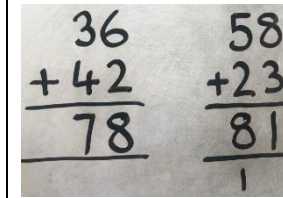
Use 10 frames to support mental methods, including bonds to the next 10.



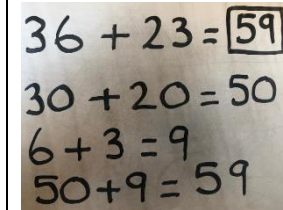
Use drawn representations of Dienes to add pairs of 2-digit numbers, counting tens then ones up to the sum.



Using a number line, start at the larger addend and then count on to arrive at the sum.



Add pairs of 2-digit numbers, using the column method with carrying if required.

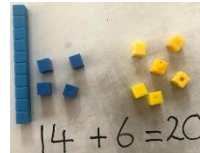
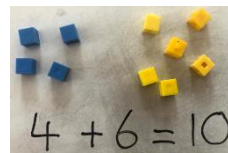


Mentally, with jottings if required, add tens and ones then recombine to find the sum.

Recall all number bonds to 10 and use this to reason bonds to 20.



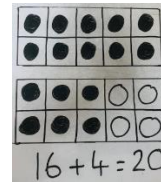
Pairs of ten frames with different-coloured counters to show each addend.



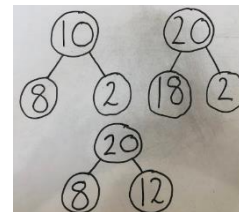
Use Dienes to show bonds to 10 and then a ten Diene added to either addend.



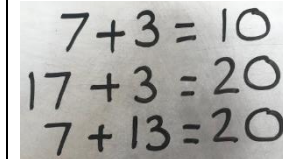
Separate 20 beads on a bead string and discuss how the bonds to 10 relate to the bonds to 20.



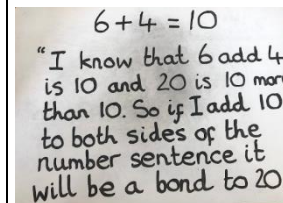
Use drawn representations of addends in pairs of ten frames to identify bonds to 20.



Present bonds to 10 in part-whole models with drawn representations of each addend and then display a bond to 20 by adding 10 to one addend.



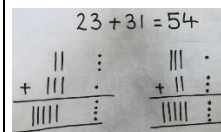
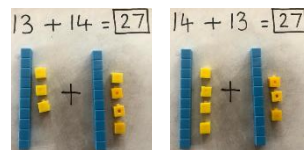
Display number bonds to 10 in number sentences and then add 10 to one addend to find a bond to 20.



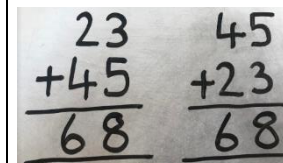
Use appropriate vocabulary to explain the link between bonds to 10 and bonds to 20.

Show that two numbers can be added in any order but not when subtracting.

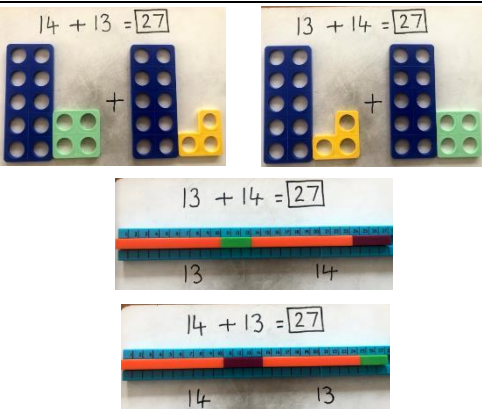
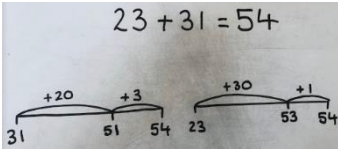
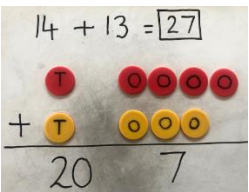
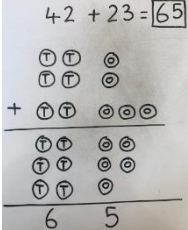
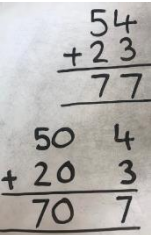
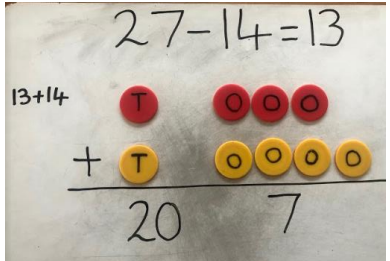
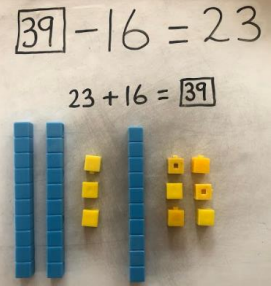
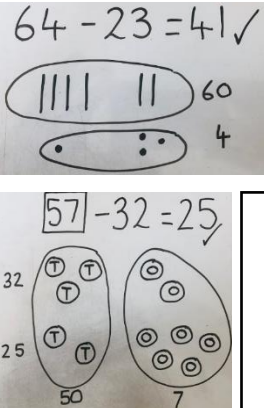
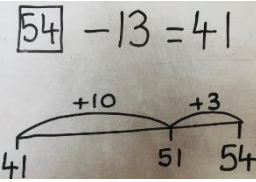
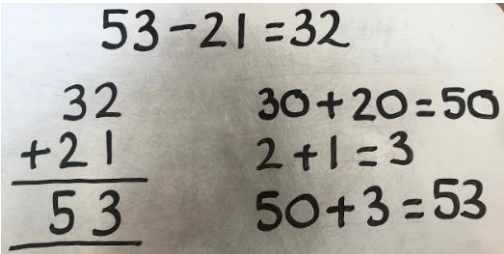
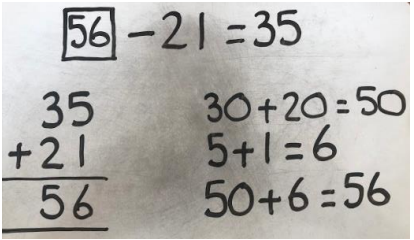
Use a range of physical objects to show how changing the order of addends does not affect the sum.



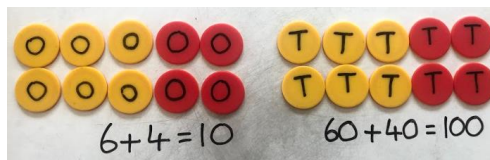
Use a range of drawn representations (including number lines) to show how changing the order of addends does not affect the sum.



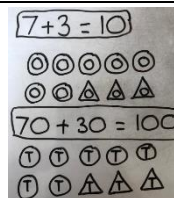
Use column addition to show how the sum is not affected by the order in which the addends are placed.

			
Add two numbers that have tens and units using column method with no carrying.	 Use physical manipulatives, which represent tens and ones, to model the column method for addition.	 Use drawn representations of tens and ones, to model the column method for addition.	 Use the column method for addition (including the expanded method) to add pairs of 2-digit numbers.
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	 Use physical manipulatives to add pairs of numbers to check subtraction number sentences.  Use physical manipulatives to add pairs of numbers to find a missing minuend in a subtraction number sentence.	 Use drawn representations of numbers to add pairs of numbers to check subtraction number sentences. Use drawn representations of numbers to find a missing minuend in a subtraction number sentence.  Use a number line to add a subtrahend to the difference to find the missing minuend.	 Use written and mental methods to add pairs of numbers to check subtraction number sentences.  Use written and mental methods to add pairs of numbers to find a missing minuend in a subtraction number sentence.

Recall all number
bonds to 10 and use
this to reason bonds
to 100.

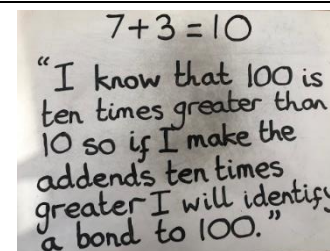
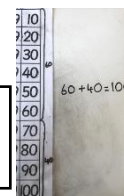


Use physical manipulatives for ones to show bonds to 10 and corresponding bonds to 100.



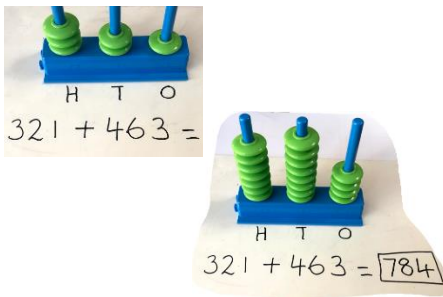
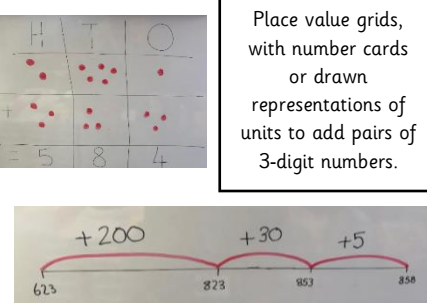
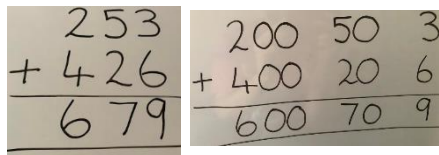
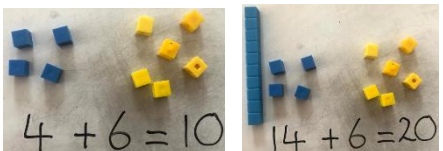
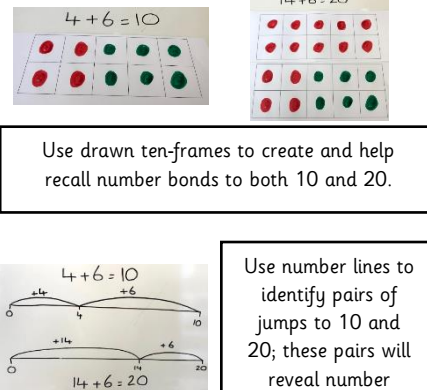
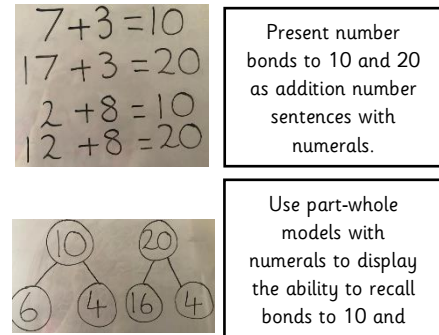
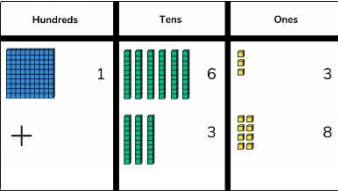
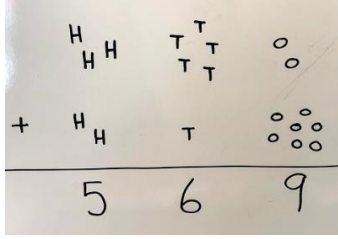
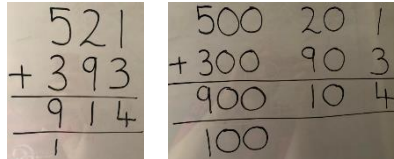
Use drawn images for ones to
show bonds to 10 and
corresponding bonds to 100.

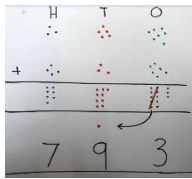
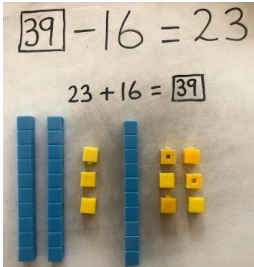
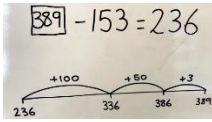
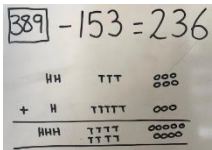
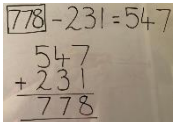
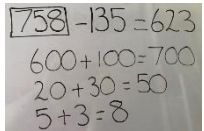
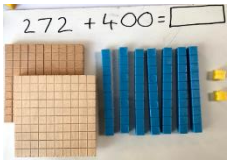
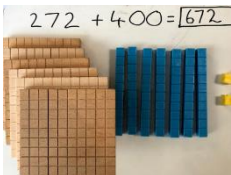
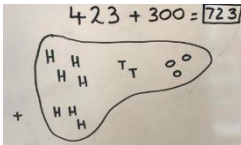
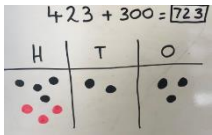
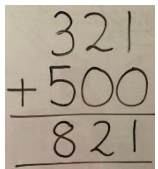
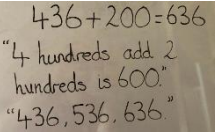
Use a 100-square to identify
bonds to 100 made up with
multiples of 10.



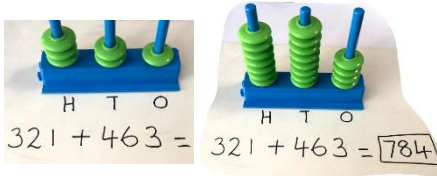
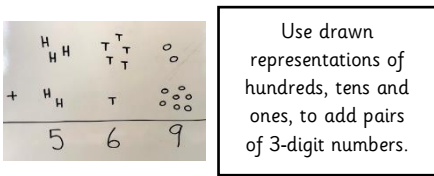
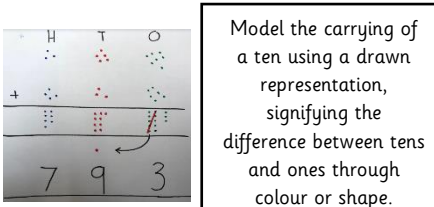
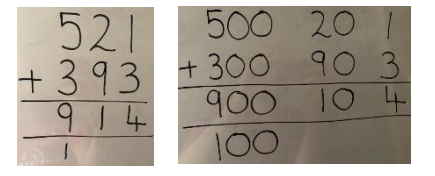
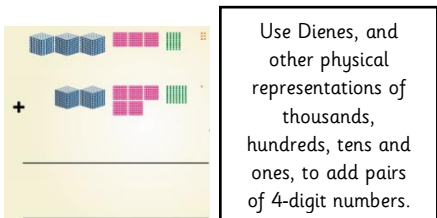
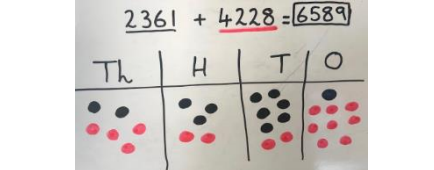
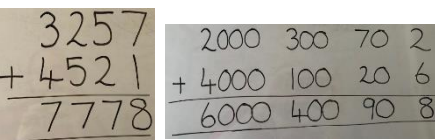
Verbally explain how number sentences for bonds to 10 can be used to identify bonds to 100 and their respective number sentences.

Year 3: Addition

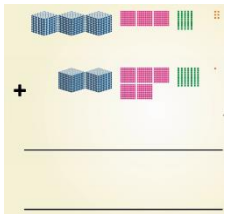
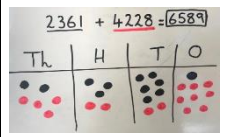
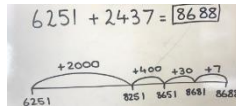
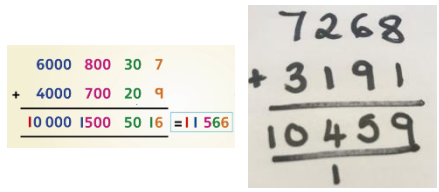
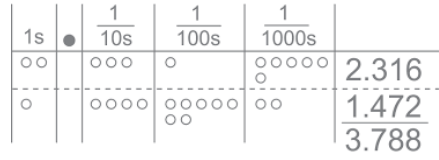
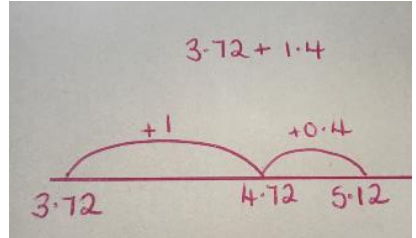
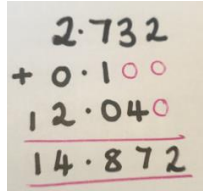
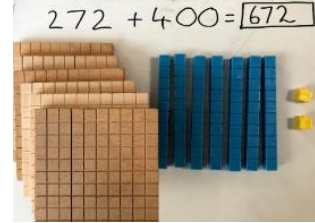
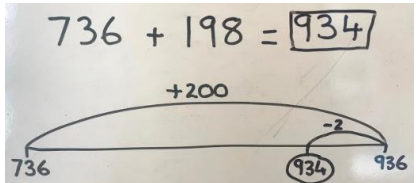
Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Add numbers with up to 3 digits where no regrouping (crossing tens) is required.	 Use Dienes, and other physical representations of hundreds, tens and ones, to add pairs of 3-digit numbers.	 Place value grids, with number cards or drawn representations of units to add pairs of 3-digit numbers. Use drawn number lines to count on from the larger addend to arrive at the sum.	 Using the column method for addition; both compact and expanded methods. Mental partitioning and recombining of hundreds, tens and ones (with jottings if required).
Recall the number bonds to 10 and 20.	 Organise sets of objects (10 and 20) into 2 groups; these groups will reveal number bonds.	 Use drawn ten-frames to create and help recall number bonds to both 10 and 20. Use number lines to identify pairs of jumps to 10 and 20; these pairs will reveal number bonds.	 Present number bonds to 10 and 20 as addition number sentences with numerals. Use part-whole models with numerals to display the ability to recall bonds to 10 and 20.
Add numbers with up to 3 digits, using the column method with carrying and exchanging.	 Use Dienes, and other physical representations of hundreds, tens and ones, to add pairs of 3-digit numbers.	 Use drawn representations of hundreds, tens and ones, to add pairs of 3-digit numbers.	 Column methods for addition (both compact and expanded). Carry tens and hundreds to the next column.

	Use physical manipulatives to model exchanging ten ones to a single ten.	 Model the carrying of a ten using a drawn representation, signifying the difference between tens and ones through colour or shape.	Using the column method to add numbers with varying amounts of digits. Ensure columns are aligned.
Solve missing number subtraction problems.	 Where the minuend is missing, use physical representations to add the difference to the subtrahend.	 Use a number line to add the subtrahend to the difference to identify the missing minuend.  Use drawn representations of numbers to add the difference to the subtrahend to find the missing minuend.	 To find a missing minuend, use the column method for addition (expanded and compact) to add the difference to the subtrahend.  Use mental methods (including partitioning and recombining) to add differences to subtrahends to find missing minuends.
Mentally add a hundreds number to any 3-digit number.	  Create 3-digit numbers using Dienes (or other physical representations of HTO) and then add multiples of hundreds.	 Use drawn representations of hundreds, tens and ones to create 3-digit numbers and then add multiples of hundreds.  Use place-value cards and tables to add hundreds to 3-digit numbers.	 Use the column method for addition to add multiples of 100 to 3-digit numbers.  Mentally add multiples of 100 by counting on in hundreds or adjusting the hundreds column (children should recognise the tens and ones will not change.).

Year 4: Addition

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Add 3-digit numbers using the column method.	 Use Dienes, and other physical representations of hundreds, tens and ones, to add pairs of 3-digit numbers. Use physical manipulatives to model exchanging ten ones to a single ten.	 Use drawn representations of hundreds, tens and ones, to add pairs of 3-digit numbers.  Model the carrying of a ten using a drawn representation, signifying the difference between tens and ones through colour or shape.	 Column methods for addition (both compact and expanded). Carry tens and hundreds to the next column. Using the column method to add numbers with varying amounts of digits. Ensure columns are aligned.
Add 4-digit numbers using the column method.	 Use Dienes, and other physical representations of thousands, hundreds, tens and ones, to add pairs of 4-digit numbers. Use physical manipulatives to model exchanging ten ones/tens/hundreds to a single ten/hundred/thousand.	 Use drawn representations of thousands, hundreds, tens and ones, to add pairs of 4-digit numbers. Model the carrying of a ten/hundred using a drawn representation, signifying the difference between hundreds, tens and ones through colour, symbol or shape.	 Column methods for addition (both compact and expanded). Carry tens and hundreds to the next column. Using the column method to add numbers with varying amounts of digits. Ensure columns are aligned.

Year 5: Addition

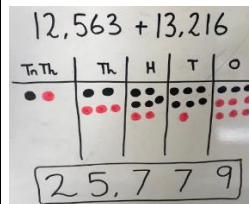
Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Add numbers with up to 4 digits using column methods.	 Add using Dienes (or other physical manipulatives).	 Use of drawn representations of ThHTO to combine addends to add 4-digit numbers.  Use of a number line to add on from the larger addend to arrive at the sum.	 Expanded and compact methods of column addition to add 4-digit numbers.
Add decimal numbers with up to 3 decimal places.	 Place-value chart with counters (or other manipulatives) to add pairs of numbers with decimal places.	 Use of a number line to add on from the larger addend to arrive at the sum.	 Expanded and compact methods of column addition to add 4-digit numbers including those with decimal places. Add pairs of numbers using the column method with different numbers of decimal places (ensure columns are aligned).
Mentally add any 2 and 3-digit numbers.	 Use of physical manipulatives to support mental workings when adding.	 Use of a number line when rounding and adjusting to mentally add 2 and 3-digit numbers.	671 + 315 = 986 Mentally (with jottings if necessary) complete 'no work' place value calculations. 374 + 434 = 808 Identifying: doubles; near doubles; and number bonds when adding mentally.

Add whole numbers with more than 4 digits, including using formal written methods.



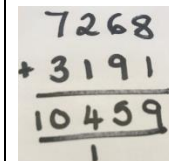
Use Dienes, and other physical representations of TnTh, Th, H, T and Os to add pairs of 5-digit numbers.

Use physical manipulatives to model exchanging ten ones/tens/hundreds/thousands to a single ten/hundred/thousand/ten thousand.

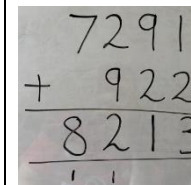


Use drawn representations of TnTh, Th, H, T and Os to add pairs of 5-digit numbers.

Model the carrying of a ten/hundred/thousand using a drawn representation, signifying the difference between thousands, hundreds, tens and ones through colour, symbol or shape.

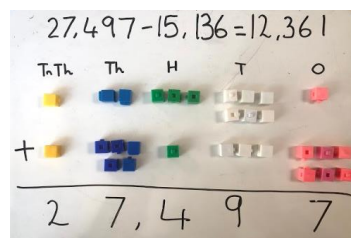


Column methods for addition (both compact and expanded). Carry tens, hundreds and thousands to the next column.

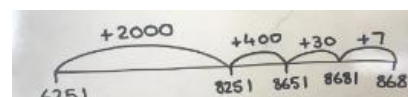


Using the column method to add numbers with varying amounts of digits. Ensure columns are aligned.

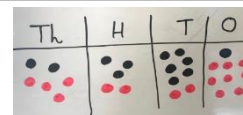
Check answers using inverse operations for subtraction.



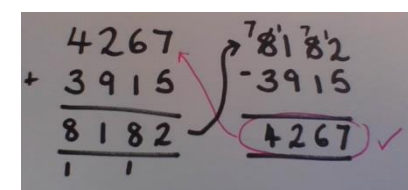
To check a subtraction, add the difference to the subtrahend using physical manipulatives for support. The sum should match the minuend.



Use a number line to add the difference to the subtrahend. The sum should be the minuend.

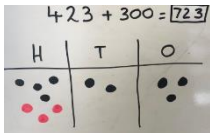
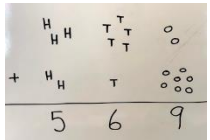
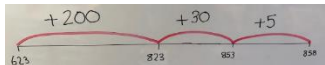
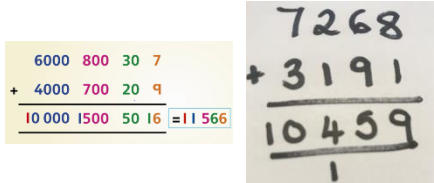
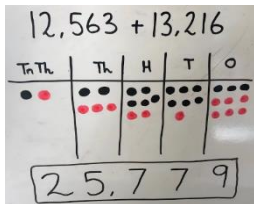
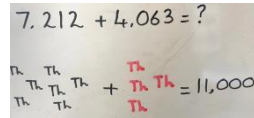
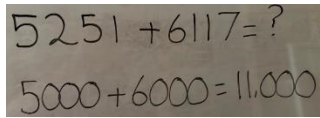


Use drawn representations of numbers to add the difference to the subtrahend. The sum should be the minuend.



Use addition and subtraction (compact column method) to check number sentences.

Year 6: Addition

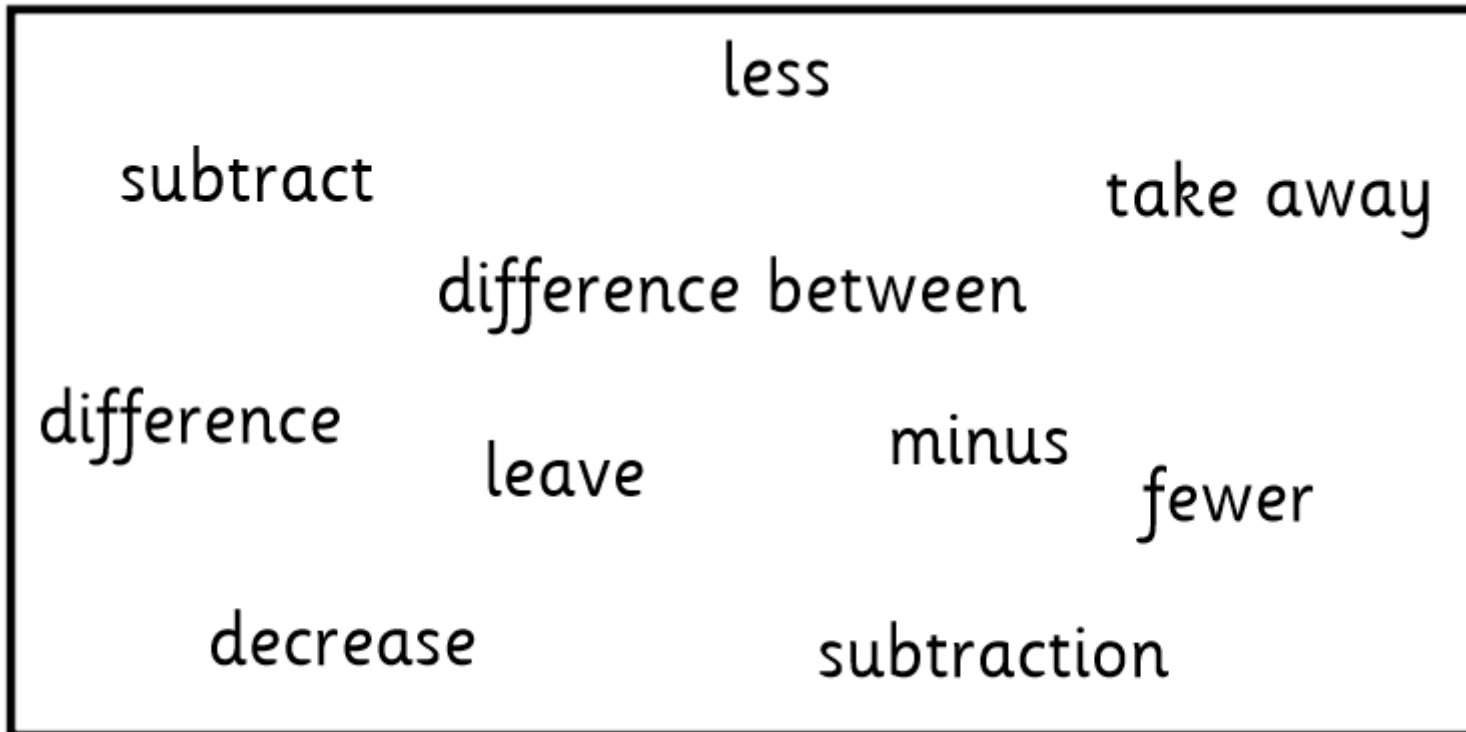
Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Mentally add any 2 and 3-digit numbers.	See previous year groups for concrete approaches to addition.	 Use pictorial representations of 2 and 3-digit numbers to support mental addition.	$436 + 201 = 637$ $436 + 200 = 636$ $636 + 1 = 637$ $671 + 315 = 986$ Rounding and adjusting – with jottings if necessary. Mental partitioning and recombining.
Add whole numbers with more than 4 digits, including using formal written methods.	See previous year groups for concrete approaches to addition.	 Use pictorial representations of numbers set out in the column method for addition.  Use number lines to count on from the greater addend to arrive at the sum.	 Column method for addition – using expanded and compact methods.
Perform mental calculations, including with mixed operations and larger numbers.	See previous year groups for concrete approaches to addition.	 Use pictorial representations of numbers to support with mental methods of addition.	$436 + 201 = 637$ $436 + 200 = 636$ $636 + 1 = 637$ $3207 + 3201 = 6408$ "3200 doubled is 6400." "7+1=8" Identifying doubles, near doubles or number bonds to support mental methods of addition.
Check answers using estimation, rounding and inverse operations for addition.	See previous year groups for concrete approaches to addition.	 Use pictorial representations for multiples of 10, 100, 1000, 10,000 etc. when rounding to estimate.	 Round numbers to the nearest appropriate multiple and add using the column method for addition to estimate and check answers.

Subtraction

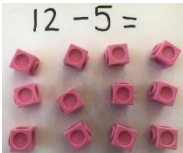
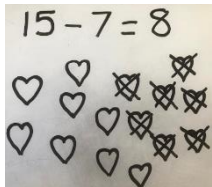
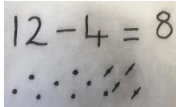
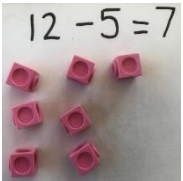
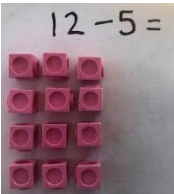
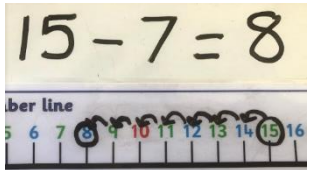
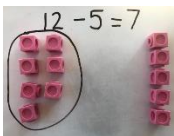
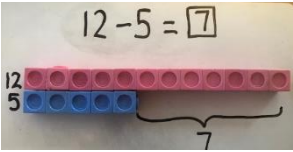
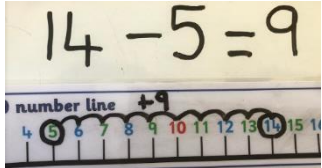


$$\begin{array}{ccccc} 8 & - & 3 & = & 5 \\ \text{Minuend} & & \text{Subtrahend} & & \text{Difference} \end{array}$$

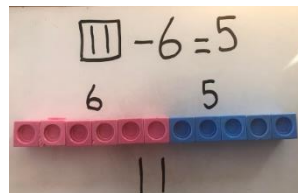
Key Vocabulary



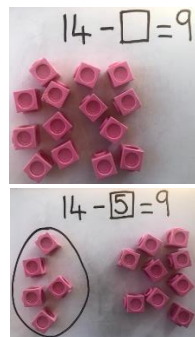
Year 1: Subtraction

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Subtract a single digit from a two-digit number up to 20.	 12 - 5 =	 15 - 7 = 8	 12 - 4 = 8
	 12 - 5 = 7	Cross out drawn objects to show what has been taken away.	Presenting subtractions in the form of number sentences with numerals.
	 12 - 5 =	 15 - 7 = 8	Reorganise the order of the number sentences, noting minuend and subtrahend cannot swap.
	 12 - 5 = 7	Use a number line to count back in ones from the minuend to arrive at the difference.	Use of language to explain the differences between numbers using appropriate vocabulary.
	 12 - 5 = 7	 14 - 5 = 9	
	Compare objects and amounts in order to find the difference.	Use a number line to count on from the subtrahend up to the minuend to find the difference.	

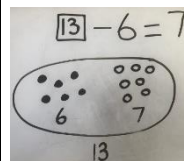
Solve missing number problems that involve subtraction from 100 using apparatus if necessary.



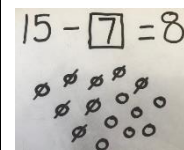
Where the minuend is missing, combine the subtrahend with the difference (represented by physical objects).



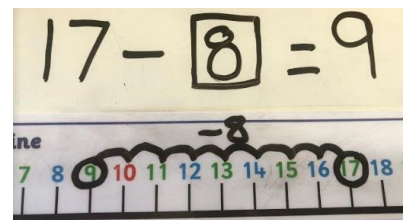
Where the subtrahend is missing, take the difference away from the minuend, which is represented using physical objects.



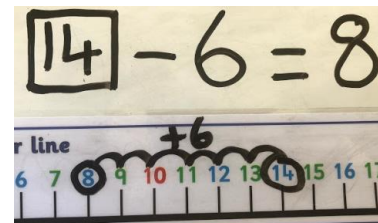
Where the minuend is missing, use pictorial representations to combine the subtrahend and the difference.



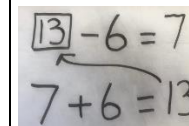
Where the subtrahend is missing, use pictorial representations to take the difference away from the minuend.



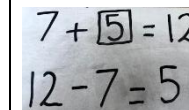
Use a number line to count back from the minuend to the difference to discover the subtrahend which was subtracted.



Use a number line to count up in ones (to the value of the subtrahend) from the difference to find the original minuend.

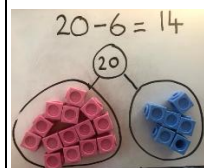


Where the minuend is missing, use the inverse operation to combine the subtrahend and the difference in a number sentence.

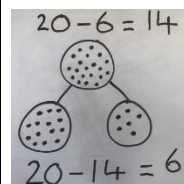


Where the subtrahend is missing, rearrange the number sentence so the difference is subtracted from the minuend.

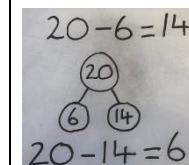
Represent and use number bonds and related subtraction facts to 20, starting to memorise.



Link to addition, using the part-whole model with physical objects to model the inverse.

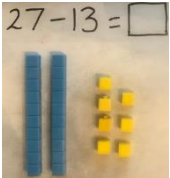
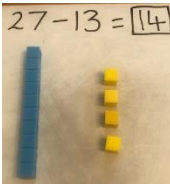
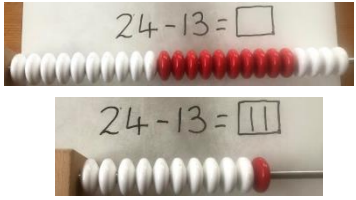
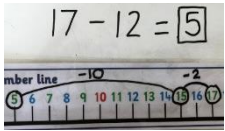
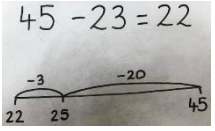
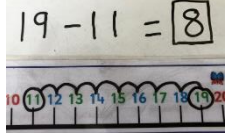
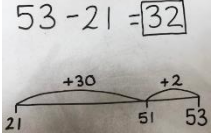
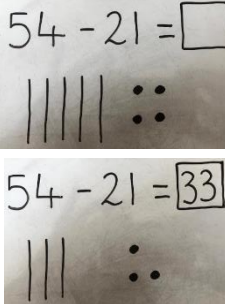
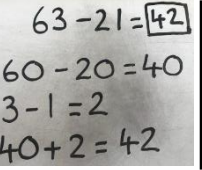
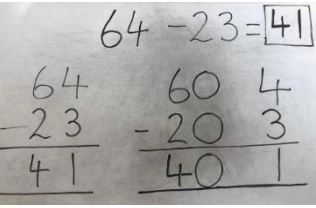
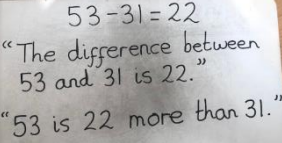
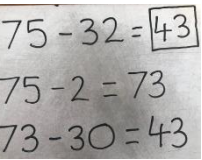
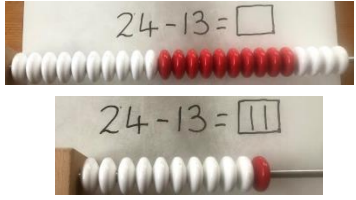
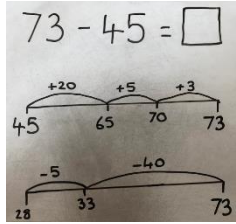
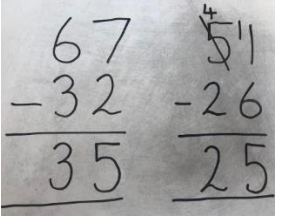


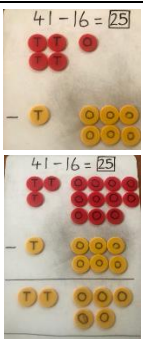
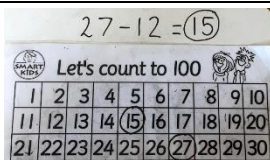
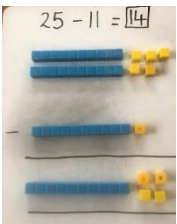
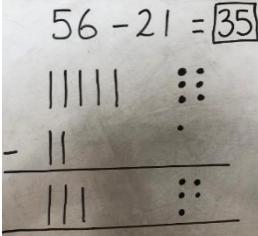
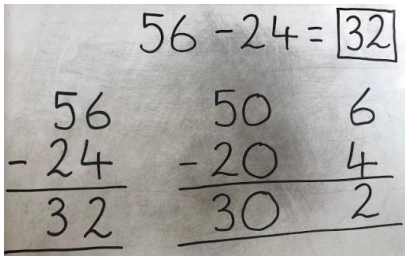
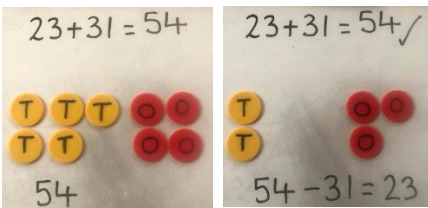
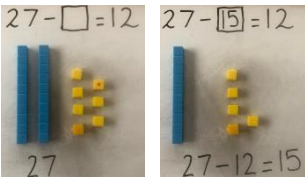
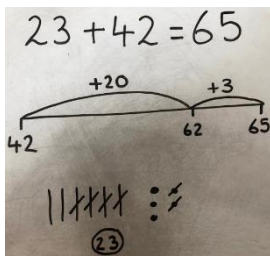
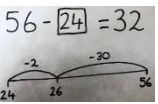
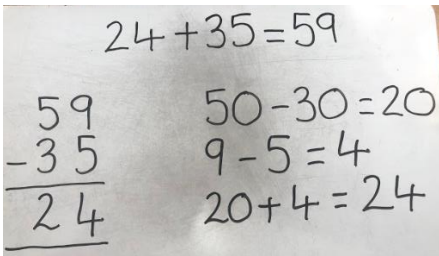
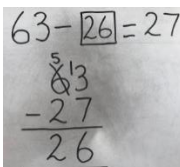
Use pictorial representations to show the parts and the whole.



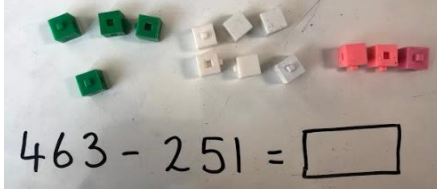
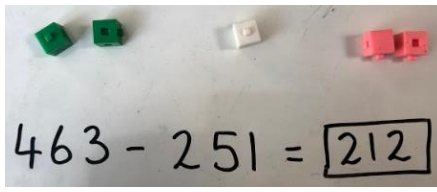
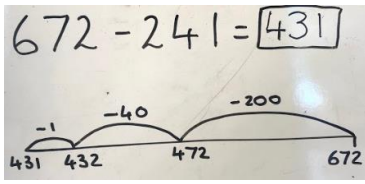
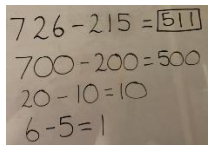
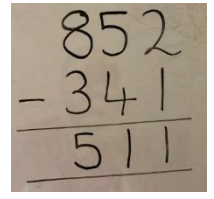
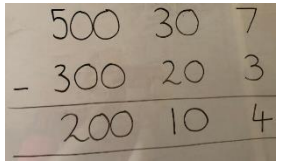
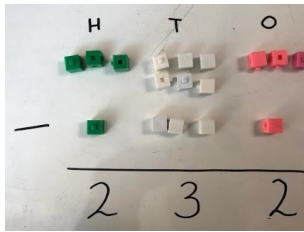
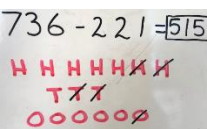
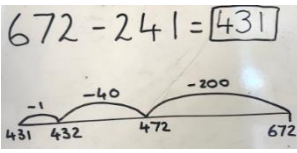
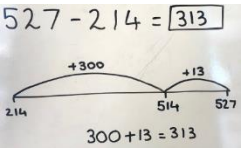
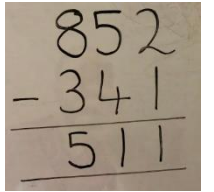
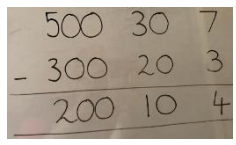
Move to using numerals within the part-whole model to model the inverse.

Year 2: Subtraction

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Subtract two 2-digit numbers where no regrouping is required (not crossing tens).	    Display the minuend using a bead string and then subtract the ones then tens of the subtrahend to arrive at the difference.	  Use number lines, both marked and self-produced, to subtract by taking away (counting back) from the minuend.   Use number lines, both marked and self-produced, to count on from the subtrahend to the minuend to find the difference.  Use drawn representations of Dienes (or other methods of showing tens and ones) and subtract the subtrahend.	 Partition numbers into tens and ones, subtract tens and ones from one another and then recombine.  Use the column method for subtraction, including the expanded method.  Use appropriate vocabulary when describing subtraction facts.  Use mental methods, with jottings if required to subtract 2-digit numbers (counting back to the nearest multiple of 10).
Subtract 2 two-digit numbers using an efficient strategy, explaining verbally.	 Use a bead string to create the minuend and then subtract the subtrahend to arrive at the difference.	 Use number lines to either count on from the subtrahend to the minuend (finding the difference) or back from the minuend (taking away).	 Subtract pairs of 2-digit numbers using the column method for subtraction, including examples where exchanging is required.

	 Display the subtraction in the column method, including exchanging a ten for ten ones if necessary.	 Count on or back to find the difference using a 100-square.	
Subtract two numbers that have tens and units using the column method with no exchanging.	 Use Dienes, or other objects which can represent tens and ones, to set the subtraction out in the form of the column method in order to build towards this method.	 Use drawings which can represent tens and ones, to set the subtraction out in the form of the column method in order to build towards this method.	 Use the column method for subtraction, including the expanded method.
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	 Use physical representations to subtract 1 and 2-digit numbers to check addition number sentences using the inverse.  Where the subtrahend is missing, subtract the difference from the minuend using physical representations of each number for support.	 Use drawn representations, including number lines to subtract 1 and 2-digit numbers to check addition number sentences using the inverse.  Where the subtrahend is missing, subtract the difference from the minuend using drawn representations, including number lines for support.	 Use written and mental methods (including the column method) to subtract 1 and 2-digit numbers to check addition number sentences  Where the subtrahend is missing, subtract the difference from the minuend using the column method for subtraction.

Year 3: Subtraction

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Subtract numbers with up to 3-digits where no regrouping (crossing tens) is required.	 $463 - 251 = \boxed{}$  $463 - 251 = \boxed{212}$ Create the minuend using Dienes (or other physical representations) and then take away the subtrahend to arrive at the difference.	 $672 - 241 = \boxed{431}$ Use of place value cards to display the minuend and move each column (hundreds, tens and ones) down based on the subtrahend.  $672 - 241 = \boxed{431}$ Use a number line to count on from the subtrahend to the minuend to find the difference or count back from the minuend to the value of the subtrahend.	 $726 - 215 = \boxed{511}$ Partition, subtract and then recombine three-digit numbers (mentally and with jottings as needed).  $\begin{array}{r} 852 \\ - 341 \\ \hline 511 \end{array}$ Column method (both expanded and compact) to subtract 3-digit numbers.  $\begin{array}{r} 500307 \\ - 300203 \\ \hline 200104 \end{array}$
Subtract numbers with up to 3 digits, using the column method with carrying and exchanging.	 $\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 2 \quad 3 \quad 2 \end{array}$ Use physical representations of hundreds, tens and ones to create the number sentence in a column format. Model exchanging a ten/hundred into the ones/tens column.	 $736 - 221 = \boxed{515}$ Use drawn representations to take away (crossing off) and draw arrows to model exchanging between columns.  $672 - 241 = \boxed{431}$ Use a number line to count back from the minuend to the value of the subtrahend to arrive at the difference.  $527 - 214 = \boxed{313}$ Use a number line to count on from the subtrahend to the minuend to find the difference.	 $\begin{array}{r} 852 \\ - 341 \\ \hline 511 \end{array}$ Column method (both expanded and compact) to subtract 3-digit numbers.  $\begin{array}{r} 500307 \\ - 300203 \\ \hline 200104 \end{array}$

Solve missing number addition and subtraction problems.

$$233 + \boxed{231} = 464$$

Where an addend is missing, subtract the known addend from the sum using physical manipulatives for support.

$$664 - \boxed{431} = 233$$

Where the subtrahend is missing, use physical manipulatives to subtract the difference from the minuend.

$$326 + \boxed{413} = 739$$

Use number lines to subtract numbers which will reveal missing addends (sum – known addend) and subtrahends (minuend – difference).

Use drawn representations to subtract numbers which will reveal missing addends (sum – known addend) and subtrahends (minuend – difference).

$$436 + \boxed{512} = 948$$

Where an addend is missing, subtract the known addend from the sum using the column method for subtraction or suitable mental methods.

$$784 - \boxed{463} = 321$$

Where the subtrahend is missing, use the column method or appropriate mental methods to subtract the difference from the minuend.

Mentally subtract a hundreds number from any 3-digit number.

$$436 - 200 = \boxed{236}$$

Create 3-digit numbers using Dienes (or other physical representations of HTO) and then take away multiples of hundreds.

$$739 - 326$$

Use drawn representations of hundreds, tens and ones to create 3-digit numbers and then take away (cross out) multiples of hundreds.

$$431 - 200 = \boxed{231}$$

Use place-value cards and tables to take away hundreds from 3-digit numbers.

$$723 - 400 = \boxed{323}$$

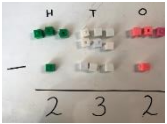
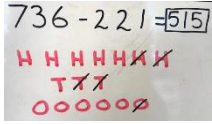
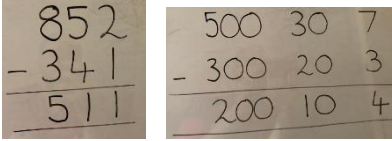
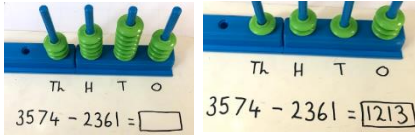
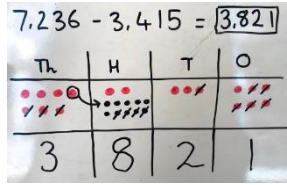
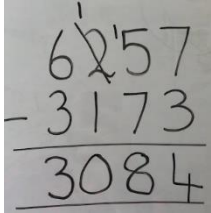
Use the column method for subtraction to subtract multiples of 100 from 3-digit numbers.

$$639 - 400 = \boxed{239}$$

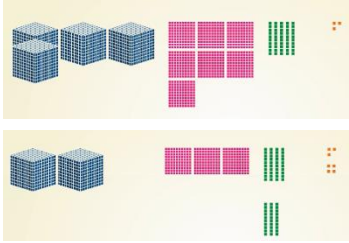
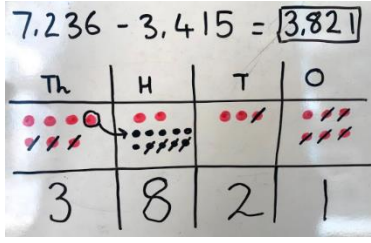
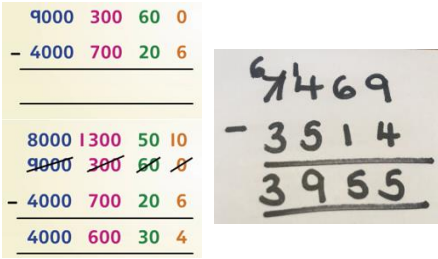
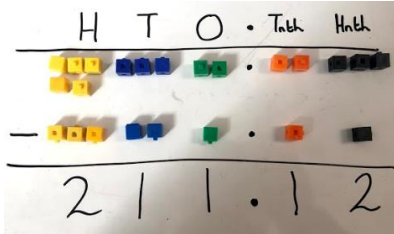
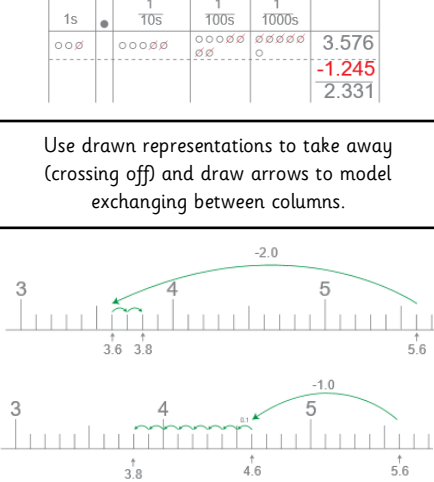
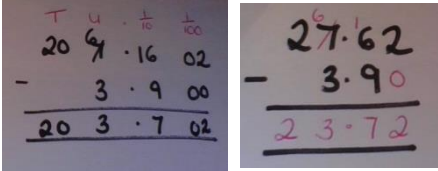
"639, 539, 439, 339, 239."

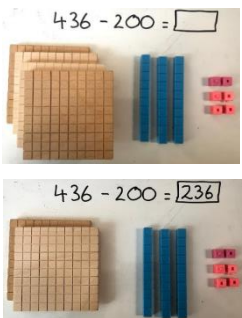
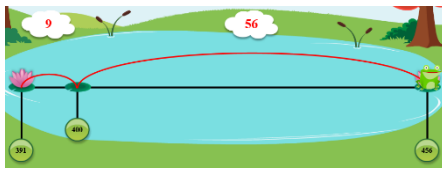
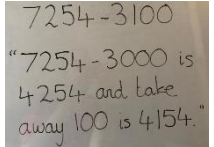
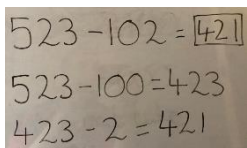
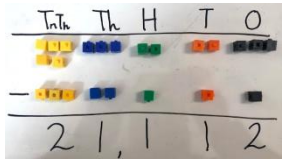
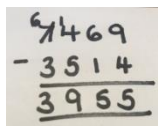
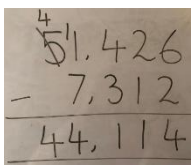
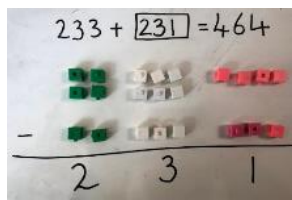
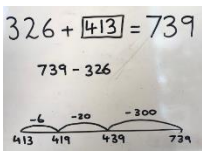
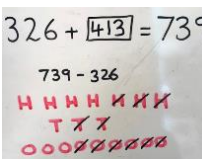
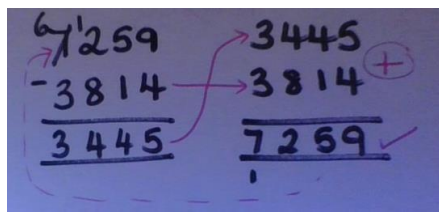
Mentally subtract multiples of 100 by counting back in hundreds or adjusting the hundreds column (children should recognise the tens and ones will not change.).

Year 4: Subtraction

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Subtract 3-digit numbers using the column method.	 Use physical representations of hundreds, tens and ones to create the number sentence in a column format.	 Use drawn representations to take away (crossing off) and draw arrows to model exchanging between columns.	 Column method (both expanded and compact) to subtract 3-digit numbers.
Subtract 4-digit numbers using column methods.	 Use physical representations of thousands, hundreds, tens and ones to create the number sentence in a column format. Model exchanging a ten/hundred into the ones/tens column.	 Use drawn representations to take away (crossing off) and draw arrows to model exchanging between columns.	 Column method (both expanded and compact) to subtract 4-digit numbers.

Year 5: Subtraction

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Subtract numbers with up to 4 digits using column methods.	 Use physical representations of thousands, hundreds, tens and ones to create the number sentence in a column format. Model exchanging a ten/hundred into the ones/tens column.	 Use drawn representations to take away (crossing off) and draw arrows to model exchanging between columns.	 Column method (both expanded and compact) to subtract 4-digit numbers.
Subtract decimal numbers with up to 3 decimal places.	 Use physical manipulatives within a place value chart to create the minuend and then take away the value of the subtrahend to reveal the difference.	 Use drawn representations to take away (crossing off) and draw arrows to model exchanging between columns. Use number lines to subtract numbers with decimal places by either counting back or counting on to find the difference.	 Column method (both expanded and compact) to subtract numbers with up to 3 decimal places.

Mentally subtract any 2 and 3-digit numbers.	 Use of physical manipulatives to support mental workings when subtracting.	 Use of a number line when rounding and adjusting to mentally subtract 2 and 3-digit numbers.	 Mentally (with jottings if necessary) complete 'no work' place value calculations.  Mental rounding and adjusting when subtracting 2 and 3-digit numbers.
Subtract whole numbers with more than 4 digits, including using formal written methods.	 Use Dienes, and other physical representations of TnTh, Th, H, T and Os to subtract 5-digit numbers. Use physical manipulatives to model exchanging ten, hundreds, thousand and tens of thousands to the next column when required.	Use drawn representations of TnTh, Th, H, T and Os to subtract 5-digit numbers. (See previous year groups for imagery) Model exchanging tens, hundreds, thousands and tens of thousands using a drawn representation, signifying the difference between thousands, hundreds, tens and ones through colour, symbol or shape.	 Column methods for subtraction (both compact and expanded). Exchange tens, hundreds, thousands and tens of thousands to the next column.  Using the column method to subtract numbers with varying amounts of digits. Ensure columns are aligned.
Check answers using inverse operations for addition.	 To check an addition number sentence, subtract an addend from the sum. The difference should be the second addend.	 Use a number line to count back from the sum to the known addend. The missing number is the difference.  Use drawn representations of numbers to subtract an addend from the sum. The difference should be the second addend.	 Use subtraction and addition (compact column methods) to check number sentences.

Year 6: Subtraction

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Mentally subtract any 2 and 3-digit numbers.	See previous year groups for concrete methods when subtracting.	Use pictorial representations of 2 and 3-digit numbers to support mental subtraction. Use number lines to 'Frog' count by counting on from the subtrahend to the minuend to find the difference.	Rounding and adjusting – with jottings if necessary. 679 - 315 = 364 'No work' place value calculations.
Subtract whole numbers with more than 4 digits, including using formal written methods.	See previous year groups for concrete methods when subtracting.	Use pictorial representations of numbers set out in the column method for subtraction. Use number lines to count on from the subtrahend to the minuend or back from the minuend to the value of the subtrahend.	Column method for subtraction – using expanded and compact methods.
Perform mental calculations, including with mixed operations and large numbers.	See previous year groups for concrete methods when subtracting.	Use pictorial representations of numbers to support with mental methods of subtraction.	Rounding and adjusting – with jottings or a number line if required. 778 - 424 = 354 Identifying doubles/halves, near doubles or number bonds to support mental methods of subtraction.

Check answers using estimation, rounding and inverse operations for addition and subtraction.	See previous year groups for concrete methods when subtracting.	Use pictorial representations for multiples of 10, 100, 1000, 10,000 etc. when rounding to estimate. Use number lines to estimate with rounded numbers when subtracting.	Round numbers to the nearest appropriate multiple and subtract using the column method for subtraction to estimate and check answers. As Year 5, applying the inverse operation.
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Multiplication



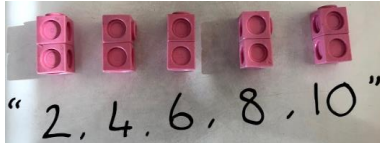
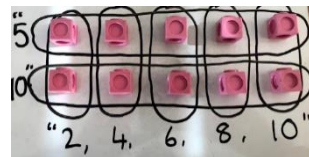
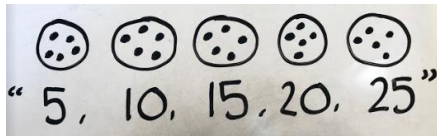
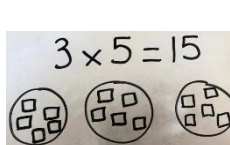
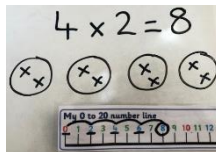
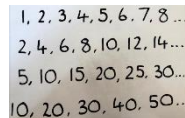
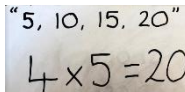
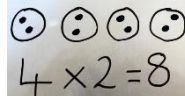
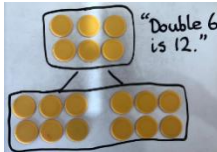
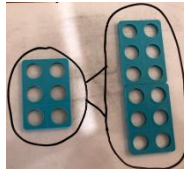
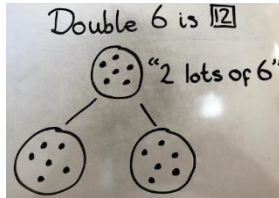
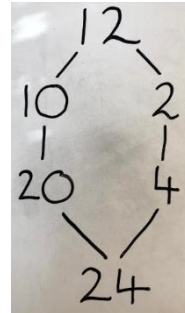
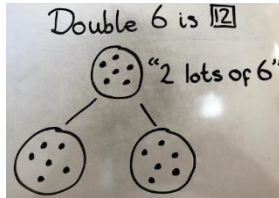
$$6 \times 3 = 18$$

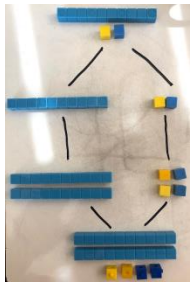
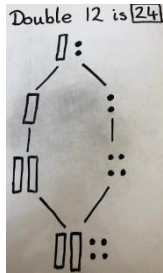
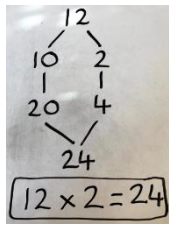
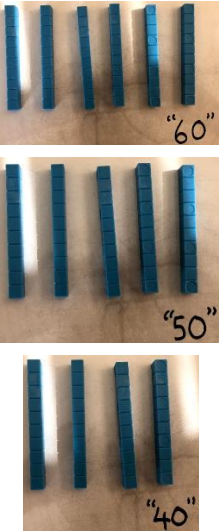
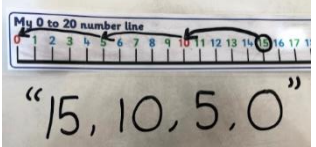
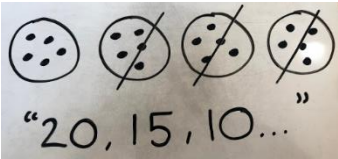
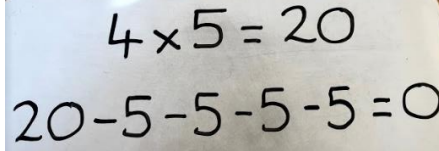
Factor (or Multiplier) Factor (or Multiplicand) Product

Key Vocabulary

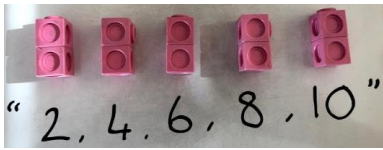
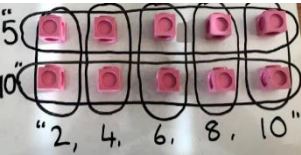
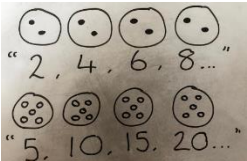
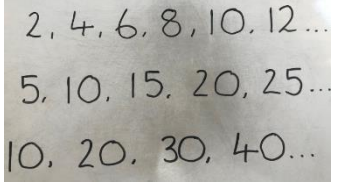
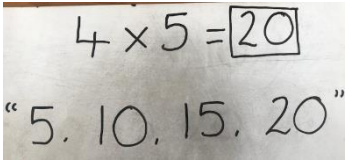
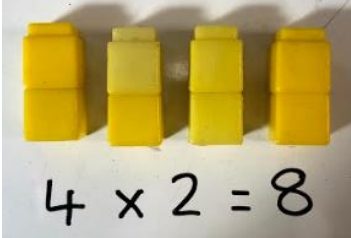
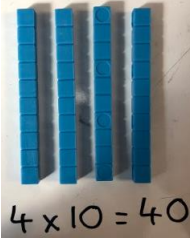
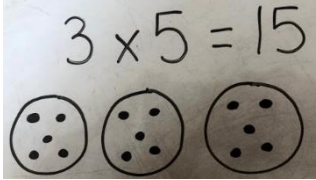
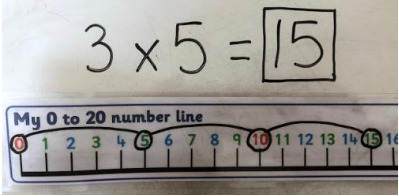
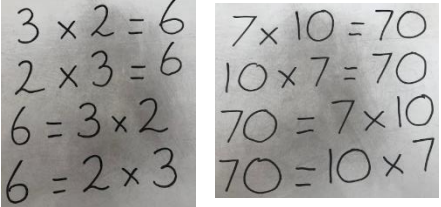
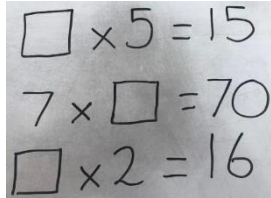
multiply	lots of	times
array	product	
multiple	multiplied by	times table
		groups of
repeated addition	multiplication	

Year 1: Multiplication

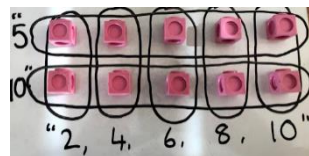
Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Count forwards in 1s, 2s, 5s and 10s up to 100 starting with 0 or 1, or any given number.	 Create groups of objects (1s, 2s, 5s, and 10s) using physical manipulatives and practise skip counting.     Organise physical objects into arrays and show how to count multiples in more than one way. Use physical manipulatives to support with repeated addition and subtraction.	 Draw groups of objects (1s, 2s, 5s and 10s) and skip count in multiples.  Use a number line to jump in multiples and count as they next multiple is landed on.  Draw images for pictorial representations of multiplication number sentences.  Use number lines and groups of pictorial representations to model repeated addition for counting in multiples.	 Write multiples of 1, 2, 5 and 10 in numerals and then practise counting these aloud.  Use the 'x' and '=' signs to write multiplication number sentences to match counting in multiples.  Build on concrete and pictorial methods by writing number sentences to describe groups of objects and pictures.
	Tell you what doubling is.	  Use a selection of physical manipulatives to demonstrate doubling. <td>  Draw pictures to show how numbers are doubled. </td> <td>  Partition a number (beyond 9), double and then recombine. </td>	 Draw pictures to show how numbers are doubled.

	 Use a selection of physical manipulatives to model partitioning, doubling and recombining.	 Double 12 is 24 Use drawn representations of numbers to model partitioning, doubling and recombining.	 Convey doubles using multiplication number sentences where numbers are multiplied by 2.
Count backwards in 1s, 2s, 5s and 10s up to 100 from any number.	 60 50 40 Create groups of objects (1s, 2s, 5s, and 10s) using physical manipulatives and count back as these are taken away.	 My 0 to 20 number line 15, 10, 5, 0 Use number lines to count back in multiples of 1, 2, 5 or 10 and use skip counting as this is done.  20, 15, 10... Draw groups of objects and take them away the child counts back in multiples.	 4 x 5 = 20 20 - 5 - 5 - 5 - 5 = 0 Write repeated subtraction number sentences which start with the product and subtract lots of the multiplicand or multiplier.

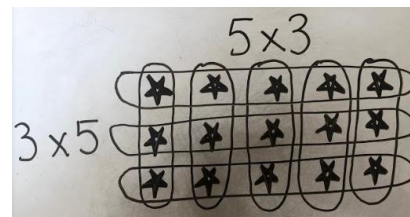
Year 2: Multiplication

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Count forwards in steps of 2, 5 and 10 from 0 and use this to solve problems.	 Create groups of objects (2s, 5s, and 10s) using physical manipulatives and practise skip counting.  Organise physical representations into arrays and show how commutativity allows us to count forwards in different ways.	 Draw groups of objects (1s, 2s, 5s, and 10s) and revise skip counting.   Use number lines, bead strings and counting sticks to revise skip counting in multiples of 2, 5 and 10.	 Write out multiples of 2, 5 and 10 using numerals.  Count in steps of 2, 5 and 10 to complete multiplication number sentences.
Recall multiplication facts for 2, 5 and 10.	  Use physical objects to represent multiplication facts and use these when revising and recalling facts for the 2, 5 and 10 times-tables.	 Use drawings to represent multiplication facts and use these when revising and recalling facts for the 2, 5 and 10 times-tables.  Use number lines to jump in multiples of 2, 5 and 10 to identify multiplication facts for these times tables.	 Use the 'x' and '=' signs to present multiplication facts for 2, 5 and 10. Model how number sentences can be rearranged.  Use missing number multiplication sentences to assist in the revision of multiplication facts for 2, 5 and 10.

Show that two numbers can be multiplied in any order but not when dividing.



Use arrays (made up using physical manipulatives) to demonstrate that the order in which numbers are multiplied does not affect the product.

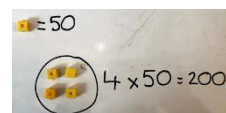
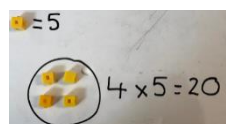


Use arrays (made up using drawn images) to demonstrate that the order in which numbers are multiplied does not affect the product.

$$\begin{aligned} 2 \times 5 &= 10 \\ 5 \times 2 &= 10 \\ 5 + 5 &= 10 \\ 2 + 2 + 2 + 2 + 2 &= 10 \end{aligned}$$

Use written multiplication number sentences (alongside repeated addition sentences) which demonstrate that the commutative law applies to multiplication.

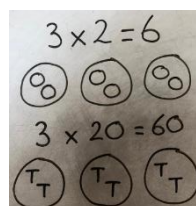
Recall and use multiplication facts for 2, 5 and 10 to calculate facts outside the times-tables.



Change the value of physical objects from ones to tens to count in multiples of 20, 50 and 100.



Use Numicon to recognise the patterns involved with multiples of 2, 5 and 10.



Change the value of drawn objects from ones to tens to count in multiples of 20, 50 and 100.



Use drawn representations to recognise the patterns involved with multiples of 2, 5 and 10.

$$\begin{aligned} 3 \times 2 &= 6 \\ 3 \times 20 &= 60 \\ 5 \times 5 &= 25 \\ 5 \times 50 &= 250 \end{aligned}$$

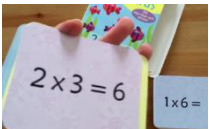
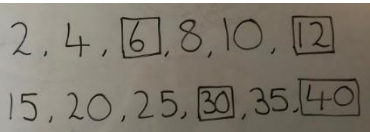
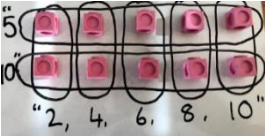
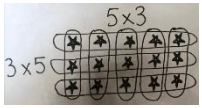
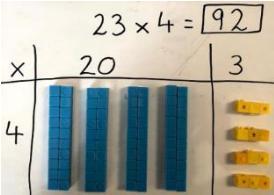
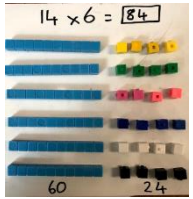
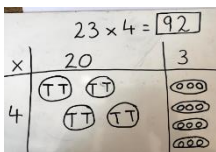
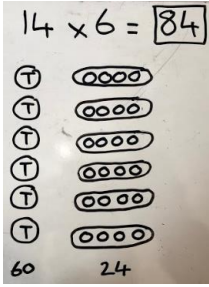
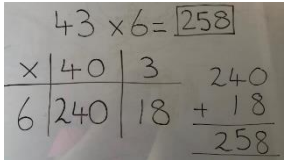
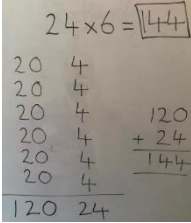
Use number sentences to show the relationship between multiples of 2, 5 and 10 and 20, 50 and 100.

72
"72 is an even number so it is a multiple of 2."

80
"80 is an even number ending in 0 so it is a multiple of 2, 5 and 10."

Use appropriate mathematical vocabulary to identify patterns which allow us to identify multiples of 2, 5 and 10.

Year 3: Multiplication

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Count forwards in steps of 2, 5 and 10 from any given number and use this to solve problems.	$\times 3$  Use of metre or counting sticks to revise counting up in multiples.	 Use of numbers lines (blank and labelled) to revise counting up in 2s, 5s and 10s.  Use flash cards to revise rapid-recall of times-tables facts.	 Fill in missing numbers in sequences of multiples. Count in steps of 2, 5 and 10 from any number and then use appropriate mathematical vocabulary to identify and continue patterns.
Answer multiplication facts for the 2, 3, 4, 5, 8, 10 and 11 times tables rapidly.	 Use physical manipulatives to create arrays which represent times-tables facts.	 Use drawn images to create arrays which represent times-tables facts. Count up in multiples on a number line to answer times-table questions.	 Use of Times Tables Rock Stars for revision and to practise rapid-recall.
Multiply a 2-digit number by a single digit using a simple grid.	 Arrange physical manipulatives into arrays within a grid to model the grid method.  Use rows to display the number of times a 2-digit number is being multiplied (14 x 6 would be 6 rows of 14) then add the counters/manipulatives.	 Insert drawn representations of tens and ones into arrays within a grid to model the grid method.  Use rows to display the number of times a 2-digit number is being multiplied (14 x 6 would be 6 rows of 14) then add the drawn representations of each value.	 Use the grid method with numerals, clearly displaying the addition next to the grid.  Partition the 2-digit numbers and use rows to show the number of times this number has been multiplied. For example, 24 x 6 would be 6 rows of 20 and 6 rows of 4.

Recall and use multiplication and division facts for 2, 5 and 10 to calculate multiplication facts outside the times-tables.

$1 = 5$



$4 \times 5 = 20$

$1 = 50$



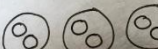
$4 \times 50 = 200$

Use physical manipulatives to represent times-tables facts and then change the value of the manipulatives to identify other known times-tables facts.

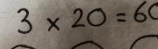


Use Numicon to recognise the patterns involved with multiples of 2, 5 and 10.

$3 \times 2 = 6$



$3 \times 20 = 60$



Change the value of drawn objects from ones to tens to count in multiples of 20, 50 and 100.



Use drawn representations to recognise the patterns involved with multiples of 2, 5 and 10.

$3 \times 2 = 6$
 $3 \times 20 = 60$
 $5 \times 5 = 25$
 $5 \times 50 = 250$

Use number sentences to show the relationship between multiples of 2, 5 and 10 and 20, 50 and 100.

72
"72 is an even number so it is a multiple of 2."

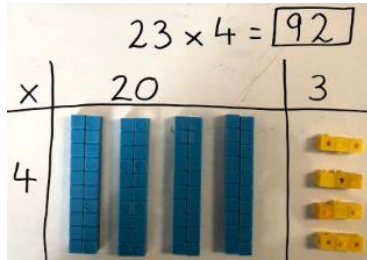
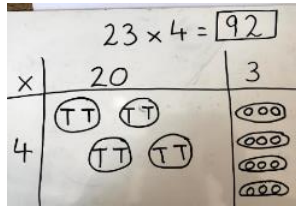
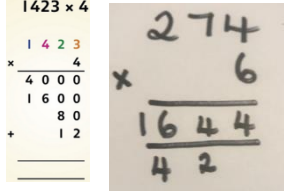
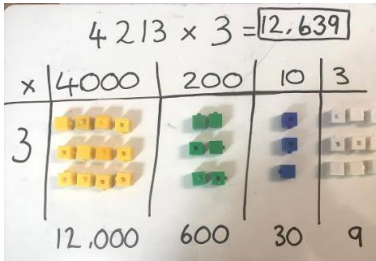
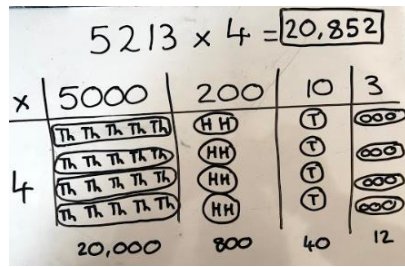
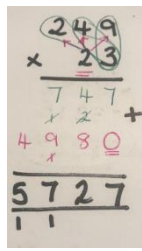
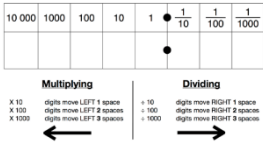
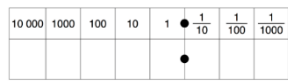
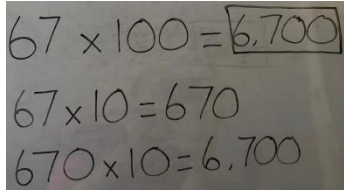
80
"80 is an even number ending in 0 so it is a multiple of 2, 5 and 10."

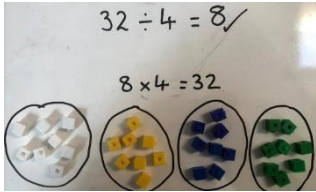
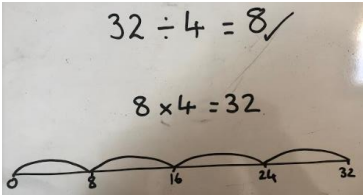
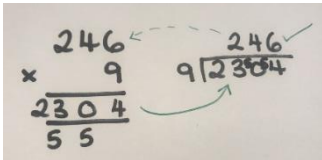
Use appropriate mathematical vocabulary to identify patterns which allow us to identify multiples of 2, 5 and 10.

Year 4: Multiplication

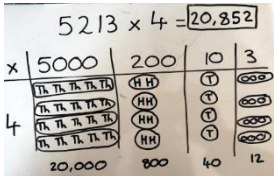
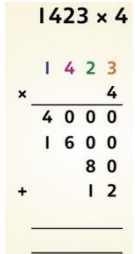
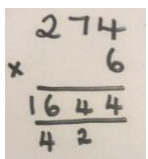
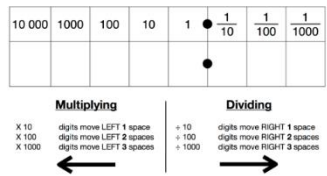
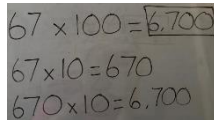
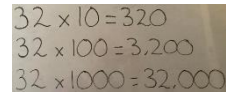
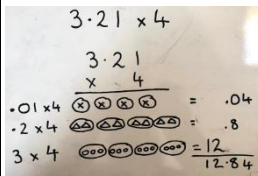
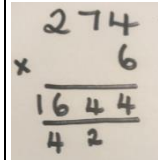
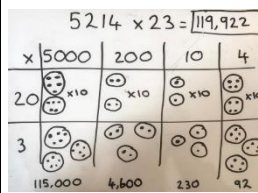
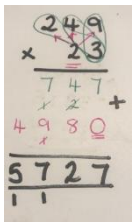
Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Multiply 2-digit and 3-digit numbers by a 1-digit number using formal written methods.	Use physical manipulatives in a column format (compact and expanded) for multiplication. Create rows of 2 and 3-digit numbers using physical representations of hundreds, tens and ones.	Use drawn images in a column format (long and short) for multiplication. Create rows of 2 and 3-digit numbers using drawn representations of hundreds, tens and ones.	Formal written short multiplication with numerals, modelling both compact and expanded methods.
Multiply 3 numbers together.	Use physical manipulatives to create arrays and then to multiply using a chosen method (grid, column, rows).	Use drawn images to create arrays and then to multiply using a chosen method (grid, column, rows).	Mental methods (with jottings) to multiply and then multiply the first product.

Year 5: Multiplication

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method																
Multiply numbers up to 4-digits by a 1-digit number using a formal written method.	 Use physical manipulatives (including place-value counters) in the grid method for multiplication.	 Use drawn representations of 4-digit numbers to multiply using the grid method or rows of the multiplier.	 Using numerals to multiply 4-digit numbers using short multiplication (compact and expanded). <table><tr><td>x</td><td>3000</td><td>200</td><td>80</td><td>4</td></tr><tr><td>5</td><td>15 000</td><td>1000</td><td>400</td><td>20</td></tr></table> Multiply 4-digit numbers using the grid method with numerals.	x	3000	200	80	4	5	15 000	1000	400	20						
x	3000	200	80	4															
5	15 000	1000	400	20															
Multiply numbers up to 4 digits by a 1-digit and 2-digit number using a formal written method.	 Use physical manipulatives (including place-value counters) in the grid method for multiplication.	 Use drawn representations of 4-digit numbers to multiply using the grid method.	 Using numerals to multiply 4-digit numbers using short and long multiplication (compact and expanded). <table><tr><td>x</td><td>30</td><td>8</td><td></td></tr><tr><td>30</td><td>900</td><td>240</td><td>= 1140</td></tr><tr><td>4</td><td>120</td><td>32</td><td>= 152</td></tr><tr><td colspan="3">Total =</td><td>1292</td></tr></table> Multiply 4-digit numbers by 2-digit numbers using the grid method with numerals.	x	30	8		30	900	240	= 1140	4	120	32	= 152	Total =			1292
x	30	8																	
30	900	240	= 1140																
4	120	32	= 152																
Total =			1292																
Multiply whole numbers by 10, 100 and 1000.	 Use counters, or other physical manipulatives, within a place-value chart to model how unit values change when numbers are multiplied by 10, 100 and 1000.	 Create and insert drawn images into place-value charts to model how unit values change when numbers are multiplied by 10, 100 and 1000. Use 2 rows, one for the original multiplier and the second for the product.	 Complete the multiplication one step at a time (repeatedly multiplying by 10).																

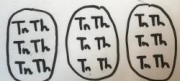
			$32 \times 10 = 320$ $32 \times 100 = 3,200$ $32 \times 1000 = 32,000$ Display multiplication facts (when multiplying by 10, 100 and 1000) in number sentences using the 'x' and '=' signs.
Check answers using inverse operations for division.	$32 \div 4 = 8 \checkmark$ $8 \times 4 = 32$  Check division number sentences by multiplying the quotient by the divisor with the use of physical manipulatives. The product should match the dividend.	$32 \div 4 = 8 \checkmark$ $8 \times 4 = 32$  Check division number sentences by multiplying the quotient by the divisor with the use of pictorial representations. The product should match the dividend.	 Check division number sentences by multiplying the quotient by the divisor with the use of formal written and mental methods (with jottings if required). The product should match the dividend.

Year 6: Multiplication

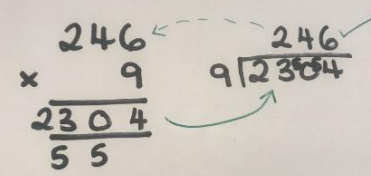
Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Multiply numbers up to 4 digits by a 1-digit number using a formal written method.	See previous year groups for column methods of multiplication.	 Use drawn representations of 4-digit numbers to multiply using the grid method or rows of the multiplier.	 Using numerals to multiply 4-digit numbers using short multiplication (compact and expanded). 
Multiply whole number by 10, 100 and 1000.	See previous year groups for column methods of multiplication.	 Create and insert drawn images into place-value charts to model how unit values change when numbers are multiplied by 10, 100 and 1000. Use 2 rows, one for the original multiplier and the second for the product.	 Complete the multiplication one step at a time (repeatedly multiplying by 10).  Display multiplication facts (when multiplying by 10, 100 and 1000) in number sentences using the 'x' and '=' signs.
Multiply 1-digit numbers with up to 2 decimal places by whole numbers.	See previous year groups for column methods of multiplication.	 Use drawn representations to multiply using the grid method and add the decimal to the answer.	 Using numerals to multiply decimal numbers using short multiplication (compact and expanded). Ensure the answer contains the same number of decimal places as the question.
Multiply 4 digit by 2 digit numbers.	See previous year groups for column methods of multiplication.	 Use drawn representations of 4-digit numbers to multiply using the grid method (long).	 Using numerals to multiply 4-digit numbers using long multiplication (compact and expanded).

Check answers using estimation, rounding and inverse operations for multiplication.

See previous year groups for column methods of multiplication.

$$29,327 \times 3 = ?$$
$$30,000 \times 3 = \boxed{90,000}$$

$$= 90,000$$

Use drawn representations of multiples of 10, 100, 1000 etc. when rounding to estimate.


$$\begin{array}{r} 246 \\ \times 9 \\ \hline 2304 \end{array}$$
$$9 \overline{)2304} \begin{array}{l} 246 \\ \hline 27 \text{ R } 2 \end{array}$$

Apply the inverse operations of multiplication and division (using formal methods) to check answers.

Division



dividend $\div$ divisor = quotient



Key Vocabulary

divide

equal sharing

share

divided into

divide into

remainder

equal groups

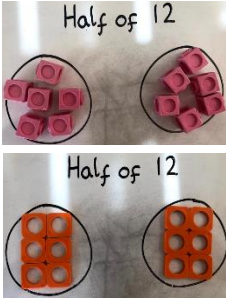
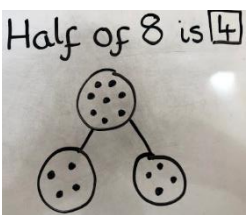
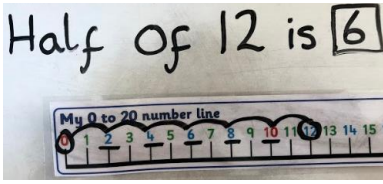
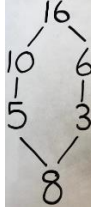
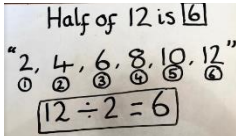
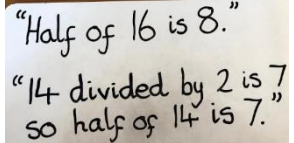
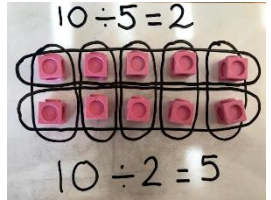
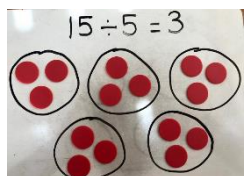
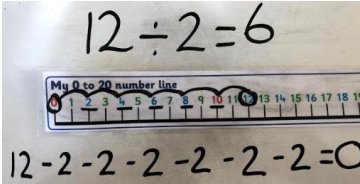
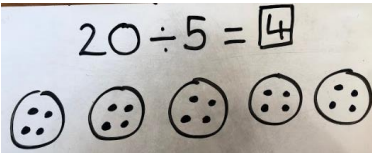
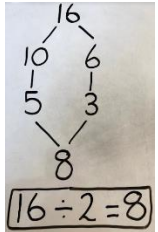
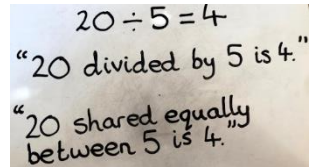
divisible by

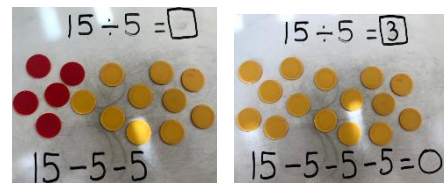
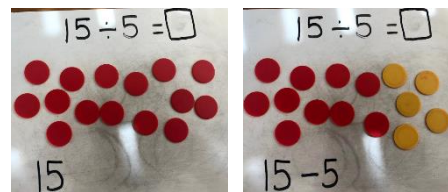
group

repeated subtraction

division

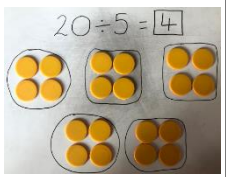
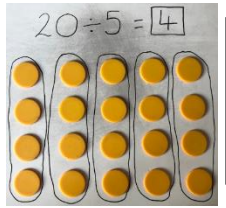
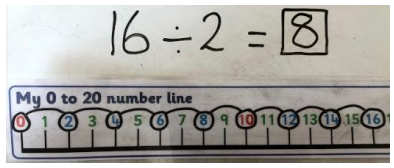
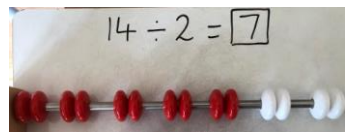
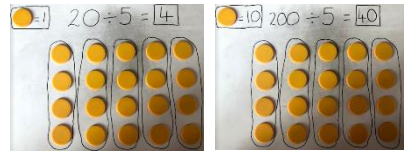
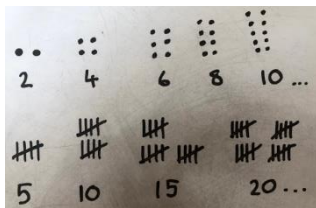
Year 1: Division

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Tell you what halving is.	 Use a selection of physical manipulatives to model sharing between two and link this to halving.	 Use part-whole models which demonstrate sharing drawn representations of numbers into two equal groups.  Use a number line to count in twos to the number being halved. The number of jumps will be half.	 Partition a number, halve each part and then recombine to halve 2-digit numbers.  Count in twos to divide by 2 and then show this as a number sentence to halve numbers.  Use of language specific to halving and dividing by 2 to find half of amounts.
Solve one-step problems up to 20 using knowledge of division of objects, graphs, charts and arrays.	 Sorting physical manipulatives into arrays to show how amounts can be divided.  Use a selection of physical manipulatives to present division as sharing (share 15 into 5 equal groups).	 Use number lines to divide through repeated subtraction.  Use drawn representations of amounts to share numbers into groups when dividing.	 Use the '÷' and '=' signs to produce division number sentences.  Use of subject-specific language for presenting division facts.



Use an amount of physical objects and carry out repeating subtraction (taking away) to divide.

Year 2: Division

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Recall division facts for 2, 5 and 10.	 $20 \div 5 = 4$ Share a number of objects between set amounts of groups to assist in the revision of division facts.  $20 \div 5 = 4$ Organise physical manipulatives into arrays to show how numbers can be divided.	 $16 \div 2 = 8$ Use a number line to count up in multiples of 2, 5 or 10 to the quotient to identify the division number sentence.  $14 \div 2 = 7$ Use a number line or bead string to start at/create the quotient then count back in 2s, 5s or 10s to 0 to identify the dividend.	$12 \div 2 = 6$ $35 \div 5 = 7$ $40 \div 10 = 4$ Use the '÷' and '=' signs to write division number sentences to revise division facts. $20 \div 5 = 4$ $20 - 5 - 5 - 5 - 5 = 0$ Present division number sentences as repeated subtractions. $35 \div 5 = 7$ "35 shared equally between 5 is 7." Use mathematical vocabulary to describe division facts for 2, 5 and 10.
Recall and use division facts for 2, 5 and 10 to calculate facts outside the times-tables.	 $20 \div 5 = 4$ $200 \div 5 = 40$ Change the value of physical objects from ones to tens to divide by 20, 50 and 100 and divide larger numbers by 2, 5 and 10.  2 4 6 8 10 two four six eight ten Use Numicon to recognise the patterns involved with multiples of 2, 5 and 10 to understand which numbers can be divided by them without a remainder.	$12 \div 2 = 6$ $120 \div 20 = 6$ Change the value of drawn objects from ones to tens to divide by 20, 50 and 100.  2 4 6 8 10 ... 5 10 15 20 ... Use drawn representations to recognise the patterns involved with multiples of 2, 5 and 10.	$45 \div 5 = 9$ $450 \div 50 = 9$ $12 \div 2 = 6$ $120 \div 20 = 6$ Use division number sentences to show the relationship between multiples of 2, 5 and 10 and 20, 50 and 100. "Any number ending with 0 is a multiple of 10." "Even numbers are all multiples of 2." "All multiples of 5 end with a 0 or a 5." Use appropriate mathematical vocabulary to identify patterns which allow us to identify multiples of 2, 5 and 10 and know what numbers can be divided by them without a remainder.

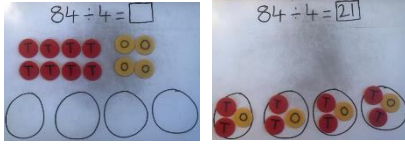
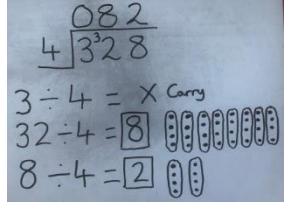
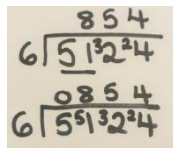
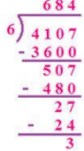
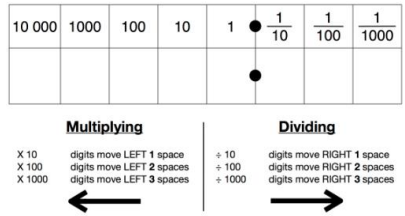
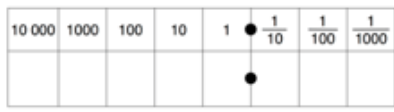
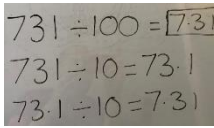
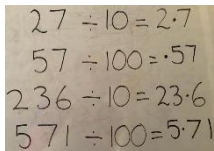
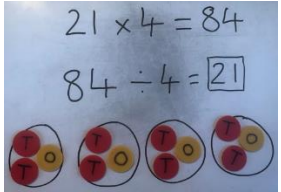
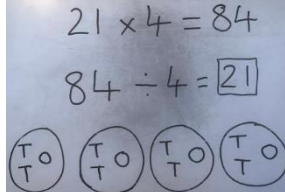
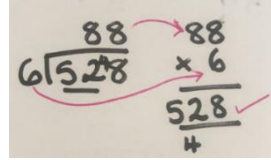
Year 3: Division

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Answer division facts for the 2, 3, 4, 5 8, 10 and 11 times tables rapidly.	Share physical manipulatives into groups to match to division number sentences. Use physical manipulatives to create arrays and match these to multiplication and division number sentences.	Share drawn representations of ones into groups to match to division number sentences. Use drawn images to create arrays and match these to multiplication and division number sentences.	Use of Times Tables Rock Stars for revision and to practise rapid-recall. Use of flash cards and missing number sentences to practise rapid recall of division facts. Match division and multiplication number sentences using knowledge of the inverse.
Recall division facts for 2, 5 and 10 to calculate multiplication facts outside the times tables.	Change the value of physical objects from ones to tens to divide by 20, 50 and 100 and divide larger numbers by 2, 5 and 10. Use Numicon to recognise the patterns involved with multiples of 2, 5 and 10 to understand which numbers can be divided by them without a remainder.	Change the value of drawn objects from ones to tens to divide by 20, 50 and 100. Use drawn representations to recognise the patterns involved with multiples of 2, 5 and 10.	Use division number sentences to show the relationship between multiples of 2, 5 and 10 and 20, 50 and 100. Use appropriate mathematical vocabulary to identify patterns which allow us to identify multiples of 2, 5 and 10 and know what numbers can be divided by them without a remainder.

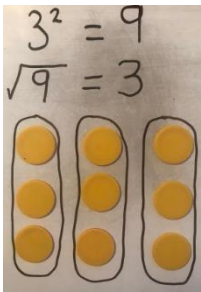
Year 4: Division

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method																																								
Divide a 1 or 2-digit number by 10 or 100.	24 ÷ 10 = <table><tr><td>T</td><td>O</td><td>.</td><td>T_{nth}</td><td>H_{nth}</td></tr><tr><td> ●● </td><td> ●●●● </td><td>.</td><td></td><td></td></tr></table> 24 ÷ 10 = 2.4 <table><tr><td>T</td><td>O</td><td>.</td><td>T_{nth}</td><td>H_{nth}</td></tr><tr><td></td><td> ●● </td><td>.</td><td> ●●●● </td><td></td></tr></table> Use place-value tables with physical manipulatives of units to model how the value changes when divided by 10 or 100.	T	O	.	T _{nth}	H _{nth}	●●	●●●●	.			T	O	.	T _{nth}	H _{nth}		●●	.	●●●●		24 ÷ 10 = <table><tr><td>T</td><td>O</td><td>.</td><td>T_{nth}</td><td>H_{nth}</td></tr><tr><td> ●● </td><td> ●●●● </td><td>.</td><td></td><td></td></tr></table> 24 ÷ 10 = 2.4 <table><tr><td>T</td><td>O</td><td>.</td><td>T_{nth}</td><td>H_{nth}</td></tr><tr><td></td><td> ●● </td><td>.</td><td> ●●●● </td><td></td></tr></table> Use place-value tables with drawn representations of units to model how the value changes when divided by 10 or 100.	T	O	.	T _{nth}	H _{nth}	●●	●●●●	.			T	O	.	T _{nth}	H _{nth}		●●	.	●●●●		18 ÷ 3 = 6 45 ÷ 5 = 9 72 ÷ 9 = 8 Use the '÷' and '=' signs to write division number sentences. Use mathematical vocabulary to explain what happens to a number when it is divided by 10 or 100 (including decimal quotients).
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Year 5: Division

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method
Divide numbers up to 4 digits by a 1-digit number using a formal written method.	 Create 4-digit numbers using physical manipulatives and share between the numbers of the divisor. Make links to short division.	 Use drawn representations and share between the same numbers of groups as the divisor. Link to formal short division.	 Use the compact method for short division (bus stop) with and without remainders.  Use the expanded method for division which will develop the skills required for long division.
Divide whole numbers by 10, 100 and 1000.	 Use counters, or other physical manipulatives, within a place-value chart to model how unit values change when numbers are divided by 10, 100 and 1000.	 Create and insert drawn images into place-value charts to model how unit values change when numbers are divided by 10, 100 and 1000. Use 2 rows, one for the original dividend and the second for the quotient.	 Complete the division one step at a time (repeatedly dividing by 10).  Write division facts (when dividing by 10, 100 and 1000) in number sentences using the '÷' and '=' signs.
Check answers using inverse operations for multiplication.	 Check multiplication number sentences by dividing the product by the multiplier with the use of physical manipulatives. The quotient should match the multiplicand.	 Check multiplication number sentences by dividing the product by the multiplier with the use of pictorial representations. The quotient should match the multiplicand.	 Check multiplication number sentences by dividing the product by the multiplier with the use of formal written and mental methods (with jottings if required). The quotient should match the multiplicand.

Use the square root sign ($\sqrt{\quad}$) accurately and confidently.



Use physical manipulatives to create arrays of square numbers to identify corresponding square roots.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

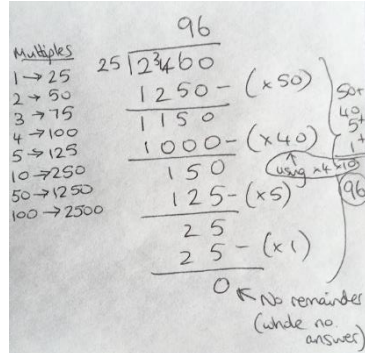
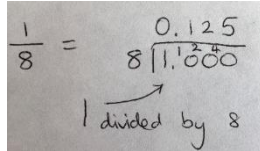
Use of a multiplication grid to identify square numbers and corresponding square roots.

$\sqrt{1} = 1$ since $1^2 = 1$
 $\sqrt{4} = 2$ since $2^2 = 4$
 $\sqrt{9} = 3$ since $3^2 = 9$
 $\sqrt{16} = 4$ since $4^2 = 16$
 $\sqrt{25} = 5$ since $5^2 = 25$
 $\sqrt{36} = 6$ since $6^2 = 36$
 $\sqrt{49} = 7$ since $7^2 = 49$
 $\sqrt{64} = 8$ since $8^2 = 64$
 $\sqrt{81} = 9$ since $9^2 = 81$
 $\sqrt{100} = 10$ since $10^2 = 100$

Using times-tables facts to recall square roots and present these as number sentences.

Use appropriate mathematical vocabulary to explain the link between square numbers and square roots.

Year 6: Division

Age-related expectation	Concrete Method	Pictorial Method	Abstract Method																																																																																																																																																																									
Divide whole numbers by 10, 100 and 1000.	See previous year groups for concrete methods when dividing.	<table><tr><td>10 000</td><td>1000</td><td>100</td><td>10</td><td>1</td><td>$\frac{1}{10}$</td><td>$\frac{1}{100}$</td><td>$\frac{1}{1000}$</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table> Create and insert drawn images into place-value charts to model how unit values change when numbers are divided by 10, 100 and 1000. Use 2 rows, one for the original dividend and the second for the quotient.	10 000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$									$731 \div 100 = 7.31$ $731 \div 10 = 73.1$ $73.1 \div 10 = 7.31$ Complete the division one step at a time (repeatedly dividing by 10). $27 \div 10 = 2.7$ $57 \div 100 = 0.57$ $236 \div 10 = 23.6$ $571 \div 100 = 5.71$ Write division facts (when dividing by 10, 100 and 1000) in number sentences using the '÷' and '=' signs.																																																																																																																																																									
10 000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$																																																																																																																																																																					
Divide numbers up to 4-digits by a 2-digit whole number using formal written methods and express remainders as whole numbers, fractions or by rounding as appropriate.	See previous year groups for concrete methods when dividing.	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr><tr><td>1</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr><tr><td>2</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td><td>22</td><td>24</td></tr><tr><td>3</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td><td>18</td><td>21</td><td>24</td><td>27</td><td>30</td><td>33</td><td>36</td></tr><tr><td>4</td><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td><td>44</td><td>48</td></tr><tr><td>5</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td><td>30</td><td>35</td><td>40</td><td>45</td><td>50</td><td>55</td><td>60</td></tr><tr><td>6</td><td>6</td><td>12</td><td>18</td><td>24</td><td>30</td><td>36</td><td>42</td><td>48</td><td>54</td><td>60</td><td>66</td><td>72</td></tr><tr><td>7</td><td>7</td><td>14</td><td>21</td><td>28</td><td>35</td><td>42</td><td>49</td><td>56</td><td>63</td><td>70</td><td>77</td><td>84</td></tr><tr><td>8</td><td>8</td><td>16</td><td>24</td><td>32</td><td>40</td><td>48</td><td>56</td><td>64</td><td>72</td><td>80</td><td>88</td><td>96</td></tr><tr><td>9</td><td>9</td><td>18</td><td>27</td><td>36</td><td>45</td><td>54</td><td>63</td><td>72</td><td>81</td><td>90</td><td>99</td><td>108</td></tr><tr><td>10</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td><td>110</td><td>120</td></tr><tr><td>11</td><td>11</td><td>22</td><td>33</td><td>44</td><td>55</td><td>66</td><td>77</td><td>88</td><td>99</td><td>110</td><td>121</td><td>132</td></tr><tr><td>12</td><td>12</td><td>24</td><td>36</td><td>48</td><td>60</td><td>72</td><td>84</td><td>96</td><td>108</td><td>120</td><td>132</td><td>144</td></tr></table> Multiplication grid can be used to support with times-tables facts beyond 12 x 12 in support of long division (chunking).	x	1	2	3	4	5	6	7	8	9	10	11	12	1	1	2	3	4	5	6	7	8	9	10	11	12	2	2	4	6	8	10	12	14	16	18	20	22	24	3	3	6	9	12	15	18	21	24	27	30	33	36	4	4	8	12	16	20	24	28	32	36	40	44	48	5	5	10	15	20	25	30	35	40	45	50	55	60	6	6	12	18	24	30	36	42	48	54	60	66	72	7	7	14	21	28	35	42	49	56	63	70	77	84	8	8	16	24	32	40	48	56	64	72	80	88	96	9	9	18	27	36	45	54	63	72	81	90	99	108	10	10	20	30	40	50	60	70	80	90	100	110	120	11	11	22	33	44	55	66	77	88	99	110	121	132	12	12	24	36	48	60	72	84	96	108	120	132	144	 Long division (chunking) using multiples outside the times-tables for support.
x	1	2	3	4	5	6	7	8	9	10	11	12																																																																																																																																																																
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Use written division methods in cases where the answer has up to 2 decimal places.	See previous year groups for concrete methods when dividing.	<table><tr><th>Decimal</th><th>Percentage</th><th>Fraction</th></tr><tr><td>0.5</td><td>50%</td><td>$\frac{1}{2}$</td></tr><tr><td>0.25</td><td>25%</td><td>$\frac{1}{4}$</td></tr><tr><td>0.75</td><td>75%</td><td>$\frac{3}{4}$</td></tr><tr><td>0.2</td><td>20%</td><td>$\frac{1}{5}$</td></tr><tr><td>0.1</td><td>10%</td><td>$\frac{1}{10}$</td></tr><tr><td>0.3</td><td>33.3%</td><td>$\frac{1}{3}$</td></tr></table> Use of FDP conversion sheet for support when converting decimals.	Decimal	Percentage	Fraction	0.5	50%	$\frac{1}{2}$	0.25	25%	$\frac{1}{4}$	0.75	75%	$\frac{3}{4}$	0.2	20%	$\frac{1}{5}$	0.1	10%	$\frac{1}{10}$	0.3	33.3%	$\frac{1}{3}$	 Long division and converting remainders to decimals using FDP conversion knowledge.																																																																																																																																																				
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Check answers using estimation, rounding and inverse operations for division.	See previous year groups for concrete methods when dividing.	If needed, use of number lines and mental methods (using known division facts with pictorial jottings) to estimate division number sentences.	The image shows handwritten mathematical work on a piece of paper. On the left, there is a long division problem: 6 is written outside a square root symbol, and 528 is written inside. A horizontal line is drawn under the 528. A red arrow points from the 88 above the 528 to the 88 in the multiplication problem on the right. On the right, there is a multiplication problem: 88 is written above a horizontal line, followed by $\times 6$. Below the line, the product 528 is written, and a red checkmark is placed to the right of the 528. Below the 528, the number 4 is written. Use of written multiplication and division methods to check answers using the inverse.
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